EQUACIONS EXPONENCIALS RESOLTES

- $3^{x-1} = \frac{3^{3x-2}}{3}$

$$3^{x-1} = \frac{3^{3x-2}}{3}$$

$$3 \cdot 3^{x-1} = 3^{3x-2}$$

$$3^{x-1+1} = 3^{3x-2} \rightarrow 3^{x} = 3^{3x-2} \rightarrow x = 3x - 2 \rightarrow x - 3x = -2 \rightarrow -2x = -2 \rightarrow \boxed{x = 1}$$

- $3^{x+1} = 81$

$$3^{x+1} = 81$$

$$3^{x+1} = 3^{4} \rightarrow x + 1 = 4 \rightarrow \boxed{x = 3}$$

- $5^{x^2-2x} = 125$

$$5^{x^2-2x} = 125$$

$$5^{x^2-2x} = 5^{3} \rightarrow x^2 - 2x = 3 \rightarrow x^2 - 2x - 3 = 0$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} = \begin{cases} \boxed{X_1 = 3} \\ \boxed{X_2 = -1} \end{cases}$$

- $2^{x^2-5x+6} = 1$

$$2^{x^2-5x+6} = 1$$

$$2^{x^2-5x+6} = 2^{0} \rightarrow x^2 - 5x + 6 = 0$$

$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2} = \begin{cases} \boxed{X_1 = 3} \\ \boxed{X_2 = 2} \end{cases}$$

- $\sqrt{a^{1-x^2}} = \frac{1}{a^4}$

$$\sqrt{a^{1-x^2}} = \frac{1}{a^4}$$

$$\left(a^{1-x^2}\right)^{\frac{1}{2}} = a^{-4}$$

$$a^{\frac{1}{2}-\frac{x^2}{2}} = a^{-4} \rightarrow \frac{1}{2} - \frac{x^2}{2} = -4 \rightarrow \frac{1}{2} - \frac{x^2}{2} = \frac{-8}{2} \rightarrow 1 - x^2 = -8 \rightarrow 1 + 8 = x^2$$

$$x^2 = 9 \rightarrow \boxed{x = \pm\sqrt{9} = \pm 3}$$

- $\left(a^x\right)^2 = \left(a^x\right)^x$

$\left(a^x\right)^2 = \left(a^x\right)^x$

$a^{2x} = a^{x^2} \rightarrow 2x = x^2 \rightarrow 2x - x^2 = 0 \rightarrow x(2-x) = 0 \rightarrow \boxed{x = 0}$

$2 - x = 0 \rightarrow \boxed{x = 2}$

- $3^{4x+1} = 19683$

$3^{4x+1} = 19683$

$3^{4x+1} = 3^9 \rightarrow 4x + 1 = 9 \rightarrow 4x = 8 \rightarrow \boxed{x = 2}$

- $2^{x^2-5x+6} = 1$

$2^{x^2-5x+6} = 1$

$2^{x^2-5x+6} = 2^0 \rightarrow x^2 - 5x + 6 = 0$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5 \pm 1}{2} =$ $\boxed{X_1 = 3}$ $\boxed{X_2 = 2}$

- $\left(10^{x-1}\right)^x = 100$

$\left(10^{x-1}\right)^x = 100$

$10^{x^2-x} = 10^2 \rightarrow x^2 - x = 2 \rightarrow x^2 - x - 2 = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} =$ $\boxed{X_1 = 2}$ $\boxed{X_2 = -1}$

- $3^x + 3^{x+1} + 3^{x+2} = 9477$

$3^x + 3^{x+1} + 3^{x+2} = 9477$

$3^x + 3 \cdot 3^x + 3^2 \cdot 3^x = 9477$

$3^x \cdot (1 + 3 + 9) = 9477$

$3^x \cdot 13 = 9477$

$3^x = \frac{9477}{13}$

$3^x = 729 \rightarrow 3^x = 3^6 \rightarrow \boxed{x = 6}$

- $3^{x-1}+3^{x}+3^{x+1}=117$

$3^{x-1}+3^{x}+3^{x+1}=117$

$3^{-1}\cdot 3^{x}+3^{x}+3\cdot 3^{x}=117$

$\frac{1}{3}\cdot 3^{x}+3^{x}+3\cdot 3^{x}=117$

$\left(\frac{1}{3}+1+3\right)\cdot 3^{x}=117$

$\frac{13}{3}\cdot 3^{x}=117\rightarrow 3^{x}=\frac{117\cdot 3}{13}\rightarrow 3^{X}=27\rightarrow 3^{X}=3^{3}\rightarrow \boxed{x=3}$

- $16^{x}+16^{1-x}-10=0$

$16^{x}+16^{1-x}-10=0$

$16^{x}+16\cdot 16^{-x}-10=0$

$16^{x}+16\cdot\frac{1}{16^{x}}-10=0$ ara fem el canvi de variable $t=16^{x}$

$t+\frac{16}{t}-10=0$

$\frac{t^{2}}{t}+\frac{16}{t}-\frac{10t}{t}=\frac{0}{t}\rightarrow t^{2}+16-10t=0\rightarrow t^{2}-10t+16=0$

$t=\frac{10\pm\sqrt{(-10)^{2}-4\cdot 1\cdot 16}}{2\cdot 1}=\frac{10\pm\sqrt{100-64}}{2}=\frac{10\pm 6}{2}=$ $t_1=8$, $t_2=2$

desfem el canvi de variable

per $t_{1}=8\rightarrow 16^{x}=8\rightarrow (2^{4})^{x}=2^{3}\rightarrow 2^{4x}=2^{3}\rightarrow 4x=3\rightarrow \boxed{x=\frac{3}{4}}$

per $t_{2}=2\rightarrow 16^{x}=2\rightarrow (2^{4})^{x}=2\rightarrow 2^{4x}=2^{1}\rightarrow 4x=1\rightarrow \boxed{x=\frac{1}{4}}$

- $5^{2x-1}=\sqrt[3]{25^{x^{2}-\frac{1}{4}}}$

$5^{2x-1}=\left(25^{x^{2}-\frac{1}{4}}\right)^{\frac{1}{3}}$

$5^{2x-1}=25^{\frac{1}{3}\cdot\left(x^{2}-\frac{1}{4}\right)}$

$5^{2x-1}=25^{\frac{x^{2}}{3}-\frac{1}{12}}$

$5^{2x-1} = \left(5^2\right)^{\frac{x^2}{3}-\frac{1}{12}}$

$5^{2x-1} = 5^{2\cdot\left(\frac{x^2}{3}-\frac{1}{12}\right)}$

$5^{2x-1} = 5^{\frac{2x^2}{3}-\frac{1}{6}} \rightarrow 2x-1 = \frac{2x^2}{3}-\frac{1}{6} \rightarrow \frac{6\cdot(2x-1)}{6} = \frac{4x^2}{3}-\frac{1}{6} \rightarrow 12x-6 = 4x^2-1$

$4x^2-12x+5=0$

$x = \frac{12\pm\sqrt{144-80}}{8} = \frac{12\pm 8}{8} = \begin{cases} x_1 = \frac{20}{8} = \boxed{\frac{5}{2}} \\ x_2 = \frac{4}{8} = \boxed{\frac{1}{2}} \end{cases}$

- $2^{2x}+2^{2x-1}+2^{2(x-1)}+2^{2x-3}+2^{2(x-2)}=1984$

$2^{2x}+2^{2x-1}+2^{2(x-1)}+2^{2x-3}+2^{2(x-2)}=1984$

$2^{2x}+2^{2x-1}+2^{2x-2}+2^{2x-3}+2^{2x-4}=1984$

$2^{2x}+2^{-1}\cdot2^{2x}+2^{-2}\cdot2^{2x}+2^{-3}\cdot2^{2x}+2^{-4}\cdot2^{2x}=1984$

$2^{2x}+\frac{1}{2}\cdot2^{2x}+\frac{1}{4}\cdot2^{2x}+\frac{1}{8}\cdot2^{2x}+\frac{1}{16}\cdot2^{2x}=1984$

$\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)\cdot2^{2x}=1984$

$\frac{31}{16}\cdot2^{2x}=1984 \rightarrow 2^{2x}=\frac{1984\cdot16}{31} \rightarrow 2^{2x}=1024 \rightarrow 2^{2x}=2^{10} \rightarrow 2x=10 \rightarrow \boxed{x=5}$

- $2^{2x}-20\cdot2^x-384=0$

$2^{2x}-20\cdot2^x-384=0$

$\left(2^x\right)^2-20\cdot2^x-384=0 \quad \text{ara fem el canvi de variable } t=2^x$

$t^2-20t-384=0$

$t=\frac{20\pm\sqrt{(-20)^2-4\cdot1\cdot(-384)}}{2\cdot1}=\frac{20\pm\sqrt{400+1536}}{2}=\frac{20\pm44}{2}= \begin{cases} t_1=32 \\ t_2=-12 \end{cases}$

desfem el canvi de variable

per $t_1=32 \rightarrow 2^x=32 \rightarrow 2^x=2^5 \rightarrow \boxed{x=5}$

per $t_2=-12 \rightarrow 2^x=-12$ que no té solució ja que 2^x sempre és >0

- $3^{2x} - 90 \cdot 3^x + 729 = 0$

$3^{2x} - 90 \cdot 3^x + 729 = 0$

$\left(3^x\right)^2 - 90 \cdot 3^x + 729 = 0$ i ara fem el canvi de variable $t = 3^x$

$t^2 - 90t + 729 = 0$

$$t = \frac{90 \pm \sqrt{(-90)^2 - 4 \cdot 1 \cdot 729}}{2 \cdot 1} = \frac{90 \pm \sqrt{(-90)^2 - 4 \cdot 1 \cdot 729}}{2 \cdot 1} = \frac{90 \pm 72}{2} = \begin{cases} t_1 = 81 \\ t_2 = 9 \end{cases}$$

Ara desfem el canvi

Per $t_1 = 81 \rightarrow 3^x = 81 \rightarrow 3^x = 3^4 \rightarrow$ $\boxed{x = 4}$

Per $t_2 = 9 \rightarrow 3^x = 9 \rightarrow 3^x = 3^2 \rightarrow$ $\boxed{x = 2}$

- $4^{x+1} + 2^{x+3} - 320 = 0$

$4^{x+1} + 2^{x+3} - 320 = 0$

$\left(2^2\right)^{x+1} + 2^{x+3} - 320 = 0$

$2^{2x+2} + 2^{x+3} - 320 = 0$

$2^2 \cdot 2^{2x} + 2^3 \cdot 2^x - 320 = 0$

$2^2 \cdot (2^x)^2 + 2^3 \cdot 2^x - 320 = 0$

$4 \cdot (2^x)^2 + 8 \cdot 2^x - 320 = 0$ ara fem el canvi de variable $t = 2^x$

$4t^2 + 8t - 320 = 0$

$$t = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 4 \cdot (-320)}}{2 \cdot 4} = \frac{-8 \pm \sqrt{5184}}{8} = \frac{-8 \pm 72}{8} = \begin{cases} t_1 = 8 \\ t_2 = -10 \end{cases}$$

desfem el canvi de variable

per $t_1 = 8 \rightarrow 2^x = 8 \rightarrow 2^x = 2^3 \rightarrow$ $\boxed{x = 3}$

per $t_2 = -10 \rightarrow 2^x = -10$ que no té solució ja que 2^x sempre és > 0

- $9^x - 6 \cdot 3^{x+1} + 81 = 0$

$9^x - 6 \cdot 3^{x+1} + 81 = 0$

$\left(3^2\right)^x - 6 \cdot 3 \cdot 3^x + 81 = 0$

$\left(3^x\right)^2 - 18 \cdot 3^x + 81 = 0$ i ara fem el canvi de variable $t = 3^x$

$t^2 - 18t + 81 = 0$

$$t=\frac{18\pm\sqrt{(-18)^2-4\cdot1\cdot81}}{2\cdot1}=\frac{18\pm\sqrt{0}}{2}=9$$

Ara desfem el canvi : per $t=9\rightarrow 3^x=9\rightarrow 3^x=3^2\rightarrow \boxed{x=2}$

- $2^{4x}-2^{2x}-12=0$

$2^{4x}-2^{2x}-12=0$

$\left(2^{2x}\right)^2-2^{2x}-12=0$ i fem el canvi de variable $2^{2x}=t$

$t^2-t-12=0$

$$t=\frac{1\pm\sqrt{(-1)^2-4\cdot1\cdot(-12)}}{2\cdot1}=\frac{1\pm7}{2}=$$

$t_1= 4$

$t_2= -3$

Desfem el canvi:

per $t_1=4\rightarrow 2^{2x}=4\rightarrow 2^{2x}=2^2\rightarrow 2x=2\rightarrow \boxed{x=1}$

per $t_1=-3\rightarrow 2^{2x}=-3$ que no té solució

Equacions exponencials que es resolen amb logaritmes

- $3^{x-1}=5$

$3^{x-1}=5$

$\log 3^{x-1}=\log 5$

$(x-1)\log 3=\log 5$

$x\cdot\log 3-\log 3=\log 5$

$x\cdot\log 3=\log 3+\log 5\rightarrow \boxed{x=\dfrac{\log 3+\log 5}{\log 3}}$

- $5\cdot2^x=3^x$

$5\cdot2^x=3^x$

$\log\left(5\cdot2^x\right)=\log 3^x$

$\log 5+\log 2^x=\log 3^x$

$\log 5+x\cdot\log 2=x\cdot\log 3$

$x\cdot\log 2-x\cdot\log 3=-\log 5$

$x\cdot(\log 2-\log 3)=-\log 5\rightarrow \boxed{x=\dfrac{-\log 5}{\log 2-\log 3}}$

- $7 \cdot 5^{x} = 3^{2x}$

$$5^{x} = \frac{3^{2x}}{7}$$

$$\log 5^{x} = \log \frac{3^{2x}}{7}$$

$$\log 5^{x} = \log 3^{2x} - \log 7$$

$$x \log 5 = 2x \log 3 - \log 7$$

$$x \log 5 - 2x \log 3 = -\log 7$$

$$x \cdot (\log 5 - 2 \log 3) = -\log 7 \rightarrow \boxed{x = \frac{-\log 7}{\log 5 - 2 \log 3}}$$