EXERCICIS SOBRE CÀLCUL DE DERIVADES II

- **Si $g(x) = [f(x)]^n \quad \rightarrow \quad g'(x) = n \cdot [f(x)]^{n-1} \cdot f'(x)$**

Exemples :

- $\left((x^3-2x)^7\right)' = 7(x^3-2x)^6 \cdot (x^3-2x)' = 7(x^3-2x)^6 \cdot (3x^2-2) = (21x^2-14)(x^3-2x)^6$
- $f(x) = \sqrt{x^2-3} \rightarrow f(x) = (x^2-3)^{\frac{1}{2}} \rightarrow f'(x) = \frac{1}{2}(x^2-3)^{-\frac{1}{2}} \cdot (x^2-3)' =$

  $\frac{1}{2}(x^2-3)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2-3}}$
- $(\cos^5 x)' = ((\cos x)^5)' = 5 \cdot (\cos x)^4 \left(\cos x\right)' = 5 \cdot (\cos x)^4 \left(-\sin x\right) = -5 \sin x \cos^4 x$
- $(\ln^7 x)' = ((\ln x)^7)' = 7 \cdot (\ln x)^6 \left(\ln x\right)' = 7 \cdot (\ln x)^6 \left(\frac{1}{x}\right) = \frac{7 \ln^6 x}{x}$
- $\left[\sin^4(5x^2-2x)\right]' = 4 \cdot (\sin(5x^2-2x))^3 \cdot (\sin(5x^2-2x))' = 4 \cdot (\sin(5x^2-2x))^3 \cdot$

  $\cos(5x^2-2x) \cdot (5x^2-2x)' = 4 \cdot (\sin(5x^2-2x))^3 \cdot \cos(5x^2-2x) \cdot$

  $(10x-2) = (40x-8) \cdot (\sin(5x^2-2x))^3 \cdot \cos(5x^2-2x)$
- $\left((5x^3e^x)^4\right)' = 4(5x^3e^x)^3 \cdot (5x^3e^x)' = 4(5x^3e^x)^3 \cdot (15x^2e^x + 5x^3e^x) = 4(15x^2e^x + 5x^3e^x)(5x^3e^x)^3$

- **Si $g(x) = \ln f(x) \quad \rightarrow \quad g'(x) = \frac{1}{f(x)} \cdot f'(x)$**

Exemples :

- $\left(\ln 10x^3\right)' = \frac{1}{10x^3}\left(10x^3\right)' = \frac{1}{10x^3} \cdot 30x^2 = \frac{3}{x}$
- $\left(\ln(5x^4-3x^2)\right)' = \frac{1}{5x^4-3x^2} \cdot (5x^4-3x^2)' = \frac{1}{5x^4-3x^2} \cdot (20x^3-6x) = \frac{20x^3-6x}{5x^4-3x^2} =$

  $\frac{\cancel{x}(20x^2-6)}{\cancel{x}(5x^3-3x)} = \frac{20x^2-6}{5x^3-3x}$
- $\left(\ln(\sin x)\right)' = \frac{1}{\sin x} \cdot (\sin x)' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot g\, x$

- $y=\ln\sqrt{x}\rightarrow y'=\left(\ln\sqrt{x}\right)'=\frac{1}{\sqrt{x}}\cdot\left(\sqrt{x}\right)'=\frac{1}{\sqrt{x}}\cdot\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2x}$
- $\left(\ln(10x\cos x)\right)'=\frac{1}{10x\cos x}\cdot(10x\cos x)'=\frac{1}{10x\cos x}\cdot(10\cos x+10x\cdot(-\sin x))=$
  $\frac{10\cos x-10x\sin x}{10x\cos x}=\frac{\cancel{10}(\cos x-x\sin x)}{\cancel{10}x\cos x}=\frac{\cos x-x\sin x}{x\cos x}$
- $f(x)=\ln\left(\frac{2x+1}{x^2}\right)\rightarrow f'(x)=\left(\ln\left(\frac{2x+1}{x^2}\right)\right)'=\frac{1}{\frac{2x+1}{x^2}}\cdot\left(\frac{2x+1}{x^2}\right)'=$
  $\frac{x^2}{2x+1}\cdot\left(\frac{2x^2-(2x+1)2x}{x^4}\right)=\frac{x^2}{2x+1}\cdot\left(\frac{2x^2-4x^2-2x}{x^4}\right)=\frac{x^2}{2x+1}\cdot\frac{-2x^2-2x}{x^4}=$
  $\frac{\cancel{x^3}(-2x-2)}{\cancel{x^3}(2x^2+x)}=\frac{-2x-2}{2x^2+x}$

- **Si $g(x)=e^{f(x)}\rightarrow g'(x)=e^{f(x)}\cdot f'(x)$**

Exemples :

- $\left(e^{5x^6}\right)'=e^{5x^6}\cdot\left(5x^6\right)'=e^{5x^6}\cdot30x^5=30x^5e^{5x^6}$
- $f(x)=e^{\sin x}\rightarrow f'(x)=\left(e^{\sin x}\right)'=e^{\sin x}\cdot(\sin x)'=e^{\sin x}\cos x$
- $f(x)=e^{\sqrt{2x+1}}\rightarrow f'(x)=\left(e^{\sqrt{2x+1}}\right)'=e^{\sqrt{2x+1}}\cdot(\sqrt{2x+1})'=e^{\sqrt{2x+1}}\cdot\frac{1}{2}(2x+1)^{-\frac{1}{2}}\cdot2=$
  $\frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}$
- $\left(e^{10x^3-5x}\right)'=e^{10x^3-5x}\cdot\left(10x^3-5x\right)'=e^{10x^3-5x}\cdot\left(30x^2-5\right)=\left(30x^2-5\right)e^{10x^3-5x}$
- $f(x)=e^{\sqrt[5]{x^2}}\rightarrow f'(x)=\left(e^{\sqrt[5]{x^2}}\right)'=e^{\sqrt[5]{x^2}}\cdot(\sqrt[5]{x^2})'=e^{\sqrt[5]{x^2}}\cdot\frac{2}{5}x^{\frac{2}{5}-1}=e^{\sqrt[5]{x^2}}\cdot\frac{2}{5}x^{\frac{-3}{5}}=\frac{2\,e^{\sqrt[5]{x^2}}}{5\sqrt[5]{x^3}}$
- $\left(e^{5\sin x-3x^2}\right)'=e^{5\sin x-3x^2}\cdot\left(5\sin x-3x^2\right)'=e^{5\sin x-3x^2}\cdot\left(5\cos x-6x\right)=\left(5\cos x-6x\right)e^{5\sin x-3x^2}$

- **Si $g(x)=\sin(f(x))\rightarrow g'(x)=\cos(f(x))\cdot f'(x)$**

Exemples :

- $\left(\sin(30x^2-5x)\right)'=\cos(30x^2-5x)\cdot(30x^2-5x)'=\cos(30x^2-5x)\cdot(60x-5)=$

  $$(60x-5)\cos(30x^2-5x)$$

- $\left(\sin(\ln x)\right)'=\cos(\ln x)\cdot(\ln x)'=\cos(\ln x)\cdot\frac{1}{x}=\frac{\cos(\ln x)}{x}$
- $\left(\sin\sqrt[4]{x^3-2}\right)'=\cos\sqrt[4]{x^3-2}\cdot(\sqrt[4]{x^3-2})'=\cos\sqrt[4]{x^3-2}\cdot\frac{1}{4}(x^3-2)^{\frac{1}{4}-1}\cdot(x^3-2)'=$

  $$\cos\sqrt[4]{x^3-2}\cdot\frac{1}{4}(x^3-2)^{-\frac{3}{4}}\cdot 3x^2=\frac{3x^2\cos\sqrt[4]{x^3-2}}{4\sqrt[4]{(x^3-2)^3}}$$

- **Si $g(x)=\cos(f(x))$ → $g'(x)=-\sin(f(x))\cdot f'(x)$**

Exemples :

- $\left(\cos(e^x)\right)'=-\sin(e^x)\cdot(e^x)'=-\sin(e^x)\cdot(e^x)=-e^x\sin(e^x)$
- $\left(\cos\sqrt[7]{x^5}\right)'=-\sin\sqrt[7]{x^5}\cdot\left(\sqrt[7]{x^5}\right)'=-\sin\sqrt[7]{x^5}\cdot\frac{5}{7}x^{\frac{5}{7}-1}=-\sin\sqrt[7]{x^5}\cdot\frac{5}{7}x^{-\frac{2}{7}}=\frac{-5\sin\sqrt[7]{x^5}}{7\sqrt[7]{x^2}}$
- $\left(\cos(3\cdot2^x-x^2)\right)'=-\sin(3\cdot2^x-x^2)\cdot(3\cdot2^x-x^2)'=-\sin(3\cdot2^x-x^2)\cdot(3\cdot2^x\cdot\ln2-2x)=$

  $$-(3\cdot2^x\cdot\ln2-2x)\sin(3\cdot2^x-x^2)$$

- $\left(\cos(\text{arctg}x)\right)'=-\sin(\text{arctg}x)\cdot(\text{arctg}x)'=-\sin(\text{arctg}x)\cdot\frac{1}{1+x^2}=-\frac{\sin(\text{arctg}x)}{1+x^2}$

- **Si $g(x)=\text{arctg}(f(x))$ → $g'(x)=\frac{1}{1+(f(x))^2}\cdot f'(x)$**

Exemples :

- $(\text{arctg}\ 3x^7)'=\frac{1}{1+\left(3x^7\right)^2}\cdot\left(3x^7\right)'=\frac{1}{1+\left(3x^7\right)^2}\cdot 21x^6=\frac{21x^6}{1+9x^{14}}$
- $(\text{arctg}\ (5x^4-\sin^2 10x))'=\frac{1}{1+(5x^4-\sin^2 10x)^2}\cdot(5x^4-\sin^2 10x)'=$

  $$\frac{1}{1+(5x^4-\sin^2 10x)^2}\cdot(20x^3-2(\sin 10x)\cdot(\cos 10x)\cdot 10)=\frac{20x^3-20\sin 10x\cos 10x}{1+(5x^4-\sin^2 10x)^2}$$

- $(arctg\,(\ln 4x))' = \dfrac{1}{1+\left(\ln 4x\right)^2}\cdot\left(\ln 4x\right)' = \dfrac{1}{1+\ln^2 4x}\cdot\dfrac{1}{\cancel{4}x}\cdot\cancel{4} = \dfrac{1}{x(1+\ln^2 4x)^2}$