

EXERCICIS SOBRE CÀLCUL DE DERIVADES II

- Si $g(x) = [f(x)]^n \rightarrow g'(x) = n \cdot [f(x)]^{n-1} \cdot f'(x)$

Exemples :

- $((x^3 - 2x)^7)' = 7(x^3 - 2x)^6 \cdot (x^3 - 2x)' = 7(x^3 - 2x)^6 \cdot (3x^2 - 2) = (21x^2 - 14)(x^3 - 2x)^6$

- $f(x) = \sqrt{x^2 - 3} \rightarrow f(x) = (x^2 - 3)^{\frac{1}{2}} \rightarrow f'(x) = \frac{1}{2}(x^2 - 3)^{-\frac{1}{2}} \cdot (x^2 - 3)' =$

$$\frac{1}{2}(x^2 - 3)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2 - 3}}$$

- $(\cos^5 x)' = ((\cos x)^5)' = 5 \cdot (\cos x)^4 (\cos x)' = 5 \cdot (\cos x)^4 (-\sin x) = -5 \sin x \cos^4 x$

- $(\ln^7 x)' = ((\ln x)^7)' = 7 \cdot (\ln x)^6 (\ln x)' = 7 \cdot (\ln x)^6 \left(\frac{1}{x}\right) = \frac{7 \ln^6 x}{x}$

- $[\sin^4(5x^2 - 2x)]' = 4 \cdot (\sin(5x^2 - 2x))^3 \cdot (\sin(5x^2 - 2x))' = 4 \cdot (\sin(5x^2 - 2x))^3 \cdot \cos(5x^2 - 2x) \cdot (5x^2 - 2x)' = 4 \cdot (\sin(5x^2 - 2x))^3 \cdot \cos(5x^2 - 2x) \cdot (10x - 2) = (40x - 8) \cdot (\sin(5x^2 - 2x))^3 \cdot \cos(5x^2 - 2x)$

- $((5x^3 e^x)^4)' = 4(5x^3 e^x)^3 \cdot (5x^3 e^x)' = 4(5x^3 e^x)^3 \cdot (15x^2 e^x + 5x^3 e^x) = 4(15x^2 e^x + 5x^3 e^x)(5x^3 e^x)^3$

- Si $g(x) = \ln f(x) \rightarrow g'(x) = \frac{1}{f(x)} \cdot f'(x)$

Exemples :

- $(\ln 10x^3)' = \frac{1}{10x^3} (10x^3)' = \frac{1}{10x^3} \cdot 30x^2 = \frac{3}{x}$

- $(\ln(5x^4 - 3x^2))' = \frac{1}{5x^4 - 3x^2} \cdot (5x^4 - 3x^2)' = \frac{1}{5x^4 - 3x^2} \cdot (20x^3 - 6x) = \frac{20x^3 - 6x}{5x^4 - 3x^2} = \frac{\cancel{x}(20x^2 - 6)}{\cancel{x}(5x^3 - 3x)} = \frac{20x^2 - 6}{5x^3 - 3x}$

- $(\ln(\sin x))' = \frac{1}{\sin x} \cdot (\sin x)' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x$

- $y = \ln \sqrt{x} \rightarrow y' = (\ln \sqrt{x})' = \frac{1}{\sqrt{x}} (\sqrt{x})' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$
- $(\ln(10x \cos x))' = \frac{1}{10x \cos x} \cdot (10x \cos x)' = \frac{1}{10x \cos x} \cdot (10 \cos x + 10x \cdot (-\sin x)) = \frac{10 \cos x - 10x \sin x}{10x \cos x} = \frac{\cancel{10}(\cos x - x \sin x)}{\cancel{10}x \cos x} = \frac{\cos x - x \sin x}{x \cos x}$
- $f(x) = \ln\left(\frac{2x+1}{x^2}\right) \rightarrow f'(x) = \left(\ln\left(\frac{2x+1}{x^2}\right)\right)' = \frac{1}{\frac{2x+1}{x^2}} \cdot \left(\frac{2x+1}{x^2}\right)' = \frac{x^2}{2x+1} \cdot \left(\frac{2x^2 - (2x+1)2x}{x^4}\right) = \frac{x^2}{2x+1} \cdot \left(\frac{2x^2 - 4x^2 - 2x}{x^4}\right) = \frac{x^2}{2x+1} \cdot \frac{-2x^2 - 2x}{x^4} = \frac{\cancel{x^2}(-2x-2)}{\cancel{x^2}(2x^2+x)} = \frac{-2x-2}{2x^2+x}$

• Si $g(x) = e^{f(x)} \rightarrow g'(x) = e^{f(x)} \cdot f'(x)$

Exemples :

- $(e^{5x^6})' = e^{5x^6} \cdot (5x^6)' = e^{5x^6} \cdot 30x^5 = 30x^5 e^{5x^6}$
- $f(x) = e^{\sin x} \rightarrow f'(x) = (e^{\sin x})' = e^{\sin x} \cdot (\sin x)' = e^{\sin x} \cos x$
- $f(x) = e^{\sqrt{2x+1}} \rightarrow f'(x) = (e^{\sqrt{2x+1}})' = e^{\sqrt{2x+1}} \cdot (\sqrt{2x+1})' = e^{\sqrt{2x+1}} \cdot \frac{1}{2} (2x+1)^{-\frac{1}{2}} \cdot 2 = \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}$
- $(e^{10x^3-5x})' = e^{10x^3-5x} \cdot (10x^3-5x)' = e^{10x^3-5x} \cdot (30x^2-5) = (30x^2-5)e^{10x^3-5x}$
- $f(x) = e^{\sqrt[5]{x^2}} \rightarrow f'(x) = (e^{\sqrt[5]{x^2}})' = e^{\sqrt[5]{x^2}} \cdot (\sqrt[5]{x^2})' = e^{\sqrt[5]{x^2}} \cdot \frac{2}{5} x^{\frac{2}{5}-1} = e^{\sqrt[5]{x^2}} \cdot \frac{2}{5} x^{-\frac{3}{5}} = \frac{2e^{\sqrt[5]{x^2}}}{5\sqrt[5]{x^3}}$
- $(e^{5\sin x-3x^2})' = e^{5\sin x-3x^2} \cdot (5\sin x-3x^2)' = e^{5\sin x-3x^2} \cdot (5\cos x-6x) = (5\cos x-6x)e^{5\sin x-3x^2}$

• Si $g(x) = \sin(f(x)) \rightarrow g'(x) = \cos(f(x)) \cdot f'(x)$

Exemples :

- $(\sin(30x^2 - 5x))' = \cos(30x^2 - 5x) \cdot (30x^2 - 5x)' = \cos(30x^2 - 5x) \cdot (60x - 5) = (60x - 5)\cos(30x^2 - 5x)$

- $(\sin(\ln x))' = \cos(\ln x) \cdot (\ln x)' = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$

- $(\sin \sqrt[4]{x^3 - 2})' = \cos \sqrt[4]{x^3 - 2} \cdot (\sqrt[4]{x^3 - 2})' = \cos \sqrt[4]{x^3 - 2} \cdot \frac{1}{4} (x^3 - 2)^{\frac{1}{4}-1} \cdot (x^3 - 2)' = \cos \sqrt[4]{x^3 - 2} \cdot \frac{1}{4} (x^3 - 2)^{-\frac{3}{4}} \cdot 3x^2 = \frac{3x^2 \cos \sqrt[4]{x^3 - 2}}{4\sqrt[4]{(x^3 - 2)^3}}$

- Si $g(x) = \cos(f(x)) \rightarrow g'(x) = -\sin(f(x)) \cdot f'(x)$

Exemples :

- $(\cos(e^x))' = -\sin(e^x) \cdot (e^x)' = -\sin(e^x) \cdot (e^x) = -e^x \sin(e^x)$

- $(\cos \sqrt[7]{x^5})' = -\sin \sqrt[7]{x^5} \cdot (\sqrt[7]{x^5})' = -\sin \sqrt[7]{x^5} \cdot \frac{5}{7} x^{\frac{5}{7}-1} = -\sin \sqrt[7]{x^5} \cdot \frac{5}{7} x^{-\frac{2}{7}} = \frac{-5 \sin \sqrt[7]{x^5}}{7\sqrt[7]{x^2}}$

- $(\cos(3 \cdot 2^x - x^2))' = -\sin(3 \cdot 2^x - x^2) \cdot (3 \cdot 2^x - x^2)' = -\sin(3 \cdot 2^x - x^2) \cdot (3 \cdot 2^x \cdot \ln 2 - 2x) = -(3 \cdot 2^x \cdot \ln 2 - 2x) \sin(3 \cdot 2^x - x^2)$

- $(\cos(\arctg x))' = -\sin(\arctg x) \cdot (\arctg x)' = -\sin(\arctg x) \cdot \frac{1}{1+x^2} = -\frac{\sin(\arctg x)}{1+x^2}$

- Si $g(x) = \arctg(f(x)) \rightarrow g'(x) = \frac{1}{1+(f(x))^2} \cdot f'(x)$

Exemples :

- $(\arctg 3x^7)' = \frac{1}{1+(3x^7)^2} \cdot (3x^7)' = \frac{1}{1+(3x^7)^2} \cdot 21x^6 = \frac{21x^6}{1+9x^{14}}$

- $(\arctg (5x^4 - \sin^2 10x))' = \frac{1}{1+(5x^4 - \sin^2 10x)^2} \cdot (5x^4 - \sin^2 10x)' =$

$$\frac{1}{1+(5x^4 - \sin^2 10x)^2} \cdot (20x^3 - 2(\sin 10x) \cdot (\cos 10x) \cdot 10) = \frac{20x^3 - 20 \sin 10x \cos 10x}{1+(5x^4 - \sin^2 10x)^2}$$

$$\blacksquare \quad (\operatorname{arctg}(\ln 4x))' = \frac{1}{1 + (\ln 4x)^2} \cdot (\ln 4x)' = \frac{1}{1 + \ln^2 4x} \cdot \frac{1}{\cancel{4}x} \cdot \cancel{4} = \frac{1}{x(1 + \ln^2 4x)^2}$$