

EXERCICIS SOBRE CÀLCUL DE DERIVADES I

- Si $f(x) = k$ amb $k \in \mathbb{R} \rightarrow f'(x) = 0$

Exemples : $(3)' = 0$

$$f(x) = \pi \rightarrow f'(x) = (\pi)' = 0$$

$$y = \frac{2}{3} \rightarrow y' = \left(\frac{2}{3}\right)' = 0$$

- Si $f(x) = x^n \rightarrow f'(x) = n \cdot x^{n-1}$

Exemples : $(x^3)' = 3x^2$

$$(x^5)' = 5x^4$$

$$(x)' = (x^1)' = 1x^0 = 1$$

$$y = x^{-2} \rightarrow y' = (x^{-2})' = -2x^{-2-1} = -2x^{-3}$$

$$\left(\frac{1}{x^3}\right)' = (x^{-3})' = -3x^{-3-1} = -3x^{-4} = -3 \cdot \frac{1}{x^4} = \frac{-3}{x^4}$$

$$f(x) = \frac{1}{x^6} \rightarrow f'(x) = \left(\frac{1}{x^6}\right)' = (x^{-6})' = -6x^{-7} = -6 \cdot \frac{1}{x^7} = \frac{-6}{x^7}$$

$$(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\left(\sqrt[3]{x^5}\right)' = \left(x^{\frac{5}{3}}\right)' = \frac{5}{3}x^{\frac{5}{3}-1} = \frac{5}{3}x^{\frac{2}{3}} = \frac{5}{3}\sqrt[3]{x^2}$$

$$y = \frac{1}{\sqrt[5]{x^6}} \rightarrow y' = \left(x^{-\frac{6}{5}}\right)' = -\frac{6}{5}x^{-\frac{6}{5}-1} = -\frac{6}{5}x^{-\frac{11}{5}} = -6 \cdot \frac{1}{\sqrt[5]{x^{11}}} = \frac{-6}{\sqrt[5]{x^{11}}}$$

- Si $f(x) = k \cdot g(x) \rightarrow f'(x) = k \cdot g'(x)$

Exemples : $(5x^6)' = 5(x^6)' = 5 \cdot 6x^5 = 30x^5$

$$f(x) = -6x^3 \rightarrow f'(x) = (-6x^3)' = -6(x^3)' = -6 \cdot 3x^2 = -18x^2$$

$$(10x^2)' = 10(x^2)' = 10 \cdot 2x^1 = 20x$$

$$y = \frac{5}{x^2} \rightarrow y' = \left(\frac{5}{x^2}\right)' = (5x^{-2})' = 5 \cdot (-2)x^{-2-1} = -10x^{-3} = -\frac{10}{x^3}$$

$$(5 \cos x)' = 5(\cos x)' = 5 \cdot (-\sin x) = -5 \sin x$$

$$\left(\frac{2}{7} \ln x\right)' = \frac{2}{7}(\ln x)' = \frac{2}{7} \left(\frac{1}{x}\right)' = \frac{2}{7x}$$

$$(\sqrt{3} \arctg x)' = \sqrt{3}(\arctg x)' = \sqrt{3} \cdot \left(\frac{1}{1+x^2}\right)' = \frac{\sqrt{3}}{1+x^2}$$

$$(5 \cdot 3^x)' = 5(3^x)' = 5 \cdot 3^x \cdot \ln 3 = 5 \cdot (\ln 3) \cdot 3^x$$

- Si $f(x) = h(x) \pm g(x) \rightarrow f'(x) = h'(x) \pm g'(x)$

Exemples : $(5x^3 + x^4)' = (5x^3)' + (x^4)' = 15x^2 + 4x^3$

$$(10x^5 - 6x + 5)' = (10x^5)' - (6x)' + (5)' = 50x^4 - 6 + 0 = 50x^4 - 6$$

$$f(x) = -3x^4 + x^{-2} + 5x \rightarrow f'(x) = (-3x^4 + x^{-2} + 5x)' =$$

$$(-3x^4)' + (x^{-2})' + (5x)' = -12x^3 - 2x^{-3} + 5$$

$$(5e^x - x^4)' = (5e^x)' - (x^4)' = 5e^x - 4x^3$$

$$(\arcsin x + 5x^4)' = (\arcsin x)' + (5x^4)' = \frac{1}{\sqrt{1-x^2}} + 20x^3$$

$$y = \sqrt{x} - \log_2 x \rightarrow y' = \left(x^{\frac{1}{2}}\right)' - (\log_2 x)' = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{x \ln 2} = \frac{1}{2\sqrt{x}} - \frac{1}{x \ln 2}$$

- Si $f(x) = h(x) \cdot g(x) \rightarrow f'(x) = h'(x) \cdot g(x) + h(x) \cdot g'(x)$

Exemples : $(5x^3 \cdot \sin x)' = (5x^3)' \cdot \sin x + 5x^3 (\sin x)' = 15x^2 \sin x + 5x^3 \cos x$

$$(x^4 e^x)' = (x^4)' \cdot e^x + x^4 (e^x)' = 4x^3 e^x + x^4 e^x = e^x (4x^3 + x^4)$$

$$f(x) = 2^x \cdot \cos x \rightarrow f'(x) = (2^x \cdot \cos x)' = (2^x)' \cdot \cos x + 2^x (\cos x)' = 2^x \ln 2 \cos x + 2^x (-\sin x) = 2^x \ln 2 \cos x - 2^x \sin x$$

$$y = \sqrt{x} \log_2 x \rightarrow y' = \left(x^{\frac{1}{2}}\right)' \cdot \log_2 x - \sqrt{x} (\log_2 x)' =$$

$$\frac{1}{2} x^{-\frac{1}{2}} \cdot \log_2 x - \sqrt{x} \frac{1}{x \ln 2} = \frac{\log_2 x}{2\sqrt{x}} - \frac{\sqrt{x}}{x \ln 2}$$

$$((5x^3 - 10x^4) \cdot \sin x)' = (5x^3 - 10x^4)' \cdot \sin x + (5x^3 - 10x^4) (\sin x)' = (15x^2 - 40x^3) \sin x + (5x^3 - 10x^4) \cos x$$

$$(\sqrt[5]{x^2} \cdot \operatorname{arc} \cot x)' = (\sqrt[5]{x^2})' \cdot \operatorname{arc} \cot x + (\sqrt[5]{x^2}) (\operatorname{arc} \cot x)' =$$

$$\left(\frac{2}{5\sqrt[5]{x^3}}\right) \operatorname{arc} \cot x + (\sqrt[5]{x^2}) \frac{-1}{1+x^2} = \frac{2 \operatorname{arc} \cot x}{5\sqrt[5]{x^3}} - \frac{\sqrt[5]{x^2}}{1+x^2}$$

- Si $f(x) = \frac{h(x)}{g(x)} \rightarrow f'(x) = \frac{h'(x) \cdot g(x) - h(x) \cdot g'(x)}{[g(x)]^2}$

Exemples : $\left(\frac{x^2 + 3x}{5x}\right)' = \frac{(x^2 + 3x)' \cdot 5x - (x^2 + 3x) \cdot (5x)'}{(5x)^2} = \frac{(2x + 3) \cdot 5x - (x^2 + 3x) \cdot 5}{25x^2} = \frac{5x^2}{25x^2} = \frac{1}{5}$

$$y = \frac{x^4}{e^x} \rightarrow y' = \left(\frac{x^4}{e^x} \right)' = \frac{(x^4)' \cdot e^x - x^4 \cdot (e^x)'}{(e^x)^2} = \frac{4x^3 e^x - x^4 e^x}{e^{2x}} = \frac{e^x (4x^3 - x^4)}{e^{2x}} = \frac{4x^3 - x^4}{e^x}$$

$$f(x) = \frac{x+2}{x-3} \rightarrow f'(x) = \left(\frac{x+2}{x-3} \right)' = \frac{1 \cdot (x-3) - (x+2) \cdot 1}{(x-3)^2} = \frac{-5}{(x-3)^2}$$

$$\left(\frac{3 \cos x}{5x^7} \right)' = \frac{(3 \cos x)' \cdot 5x^7 - 3 \cos x \cdot (5x^7)'}{(5x^7)^2} = \frac{-3 \sin x \cdot 5x^7 - 3 \cos x \cdot 35x^6}{25x^{14}} = \frac{-15x^6 (x \sin x + 7 \cos x)}{25x^{14}} = -\frac{3(x \sin x + 7 \cos x)}{5x^8}$$

$$\left(\frac{3x^4 - 2x}{5 \cdot 3^x} \right)' = \frac{(3x^4 - 2x)' \cdot 5 \cdot 3^x - (3x^4 - 2x) \cdot (5 \cdot 3^x)'}{(5 \cdot 3^x)^2} = \frac{(12x^3 - 2) \cdot 5 \cdot 3^x - (3x^4 - 2x) \cdot 5 \cdot 3^x \ln 3}{25 \cdot (3^x)^2} = \frac{5 \cdot 3^x [(12x^3 - 2) - (3x^4 - 2x) \ln 3]}{25 \cdot (3^x)^2} = \frac{12x^3 - 2 - (3x^4 - 2x) \ln 3}{5 \cdot 3^x}$$

Alguns exemples més combinant les propietats:

$$\begin{aligned} \left(\frac{3x^2 \sin x}{\sqrt[3]{x}} \right)' &= \frac{(3x^2 \sin x)' \cdot \sqrt[3]{x} - (3x^2 \sin x) (\sqrt[3]{x})'}{(\sqrt[3]{x})^2} = \frac{(6x \sin x + 3x^2 \cos x) \cdot \sqrt[3]{x} - (3x^2 \sin x) \frac{1}{3\sqrt[3]{x^2}}}{(\sqrt[3]{x})^2} = \\ &= \frac{\frac{(6x \sin x + 3x^2 \cos x) \cdot \sqrt[3]{x} \cdot 3\sqrt[3]{x^2}}{3\sqrt[3]{x^2}} - \frac{(3x^2 \sin x)}{3\sqrt[3]{x^2}}}{\sqrt[3]{x^2}} = \frac{(6x \sin x + 3x^2 \cos x) \cdot 3\sqrt[3]{x^3} - (3x^2 \sin x)}{3\sqrt[3]{x^2}} = \\ &= \frac{(6x \sin x + 3x^2 \cos x) \cdot 3x - 3x^2 \sin x}{3\sqrt[3]{x^2}} = \frac{18x^2 \sin x + 9x^3 \cos x - 3x^2 \sin x}{3\sqrt[3]{x^2}} : \frac{\sqrt[3]{x^2}}{1} = \\ &= \frac{15x^2 \sin x + 9x^3 \cos x}{3\sqrt[3]{x^2} \cdot \sqrt[3]{x^2}} = \frac{3x(5x \sin x + 3x^2 \cos x)}{3\sqrt[3]{x^4}} = \frac{\cancel{3x}(5x \sin x + 3x^2 \cos x)}{\cancel{3x} \cdot \sqrt[3]{x}} = \\ &= \frac{5x \sin x + 3x^2 \cos x}{\sqrt[3]{x}} \end{aligned}$$

$$\left(\frac{x^2 - 3x + \sqrt{x}}{2x}\right)' = \left(\frac{x^2}{2x} - \frac{3x}{2x} + \frac{\sqrt{x}}{2x}\right)' = \left(\frac{1}{2}x - \frac{3}{2} + \frac{1}{2}x^{\frac{1}{2}-1}\right)' = \left(\frac{1}{2}x - \frac{3}{2} + \frac{1}{2}x^{-\frac{1}{2}}\right)' =$$

$$\frac{1}{2} - 0 + \frac{1}{2} \cdot \left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = \frac{1}{2} - \frac{1}{4\sqrt{x^3}} = \frac{1}{2} - \frac{1}{4x\sqrt{x}}$$

$$\left(\frac{(5x^3 - 10x^4) \cdot \sin x}{e^x - 1}\right)' = \frac{\left[(5x^3 - 10x^4)' \sin x + (5x^3 - 10x^4)(\sin x)'\right](e^x - 1) - (5x^3 - 10x^4) \sin x (e^x - 1)'}{(e^x - 1)^2} =$$

$$\frac{\left[(15x^2 - 40x^3) \cdot \sin x + (5x^3 - 10x^4) \cos x\right](e^x - 1) - (5x^3 - 10x^4) e^x \sin x}{(e^x - 1)^2}$$

$$\left(5(x^2 - 3) \cdot \ln x - \frac{x+4}{x^2} \arctg x\right)' = (5(x^2 - 3) \cdot \ln x)' - \left(\frac{x+4}{x^2} \arctg x\right)' =$$

$$5 \cdot ((x^2 - 3) \cdot \ln x)' - \left(\frac{x+4}{x^2} \arctg x\right)' = 5 \cdot ((x^2 - 3)' \cdot \ln x + (x^2 - 3)(\ln x)') -$$

$$- \left[\left(\frac{x+4}{x^2}\right)' \arctg x + \frac{x+4}{x^2} (\arctg x)'\right] =$$

$$5 \cdot (2x \cdot \ln x + (x^2 - 3) \frac{1}{x}) - \left[\frac{x^2 - (x+4) \cdot 2x}{x^4} \arctg x + \frac{x+4}{x^2} \cdot \frac{1}{1+x^2}\right] =$$

$$5 \cdot (2x \cdot \ln x + (x^2 - 3) \frac{1}{x}) - \left[\frac{-x^2 - 8x}{x^4} \arctg x + \frac{x+4}{x^2(1+x^2)}\right] =$$

$$10x \ln x + \frac{5x^2 - 15}{x} + \frac{8x + x^2}{x^2} \arctg x - \frac{x+4}{x^2(1+x^2)}$$