CÀLCUL DE DERIVADES

1.TAULES

2.EXERCICIS

DERIVADES

$y = C$	$y' = 0$
$y = x^n$	$y' = n\ x^{n-1}$
$y = f(x) \pm g(x)$	$y' = f'(x) \pm g'(x)$
$y = c\ .\ f(x)$	$y' = c\ .\ f'(x)$
$y = f(x)\ .\ g(x)$	$y = f'(x)\ .\ g(x) + g'(x)\ .\ f(x)$
$y = \frac{f(x)}{g(x)}$	$y = \frac{f'(x)\ .\ g(x) - g'(x)\ .\ f(x)}{g(x)^2}$
$y = \sqrt[n]{f(x)}$	$y' = \frac{1}{n\ \sqrt[n]{f(x)^{n-1}}}\ .\ f'(x)$
$y = L\ f(x)$	$y' = \frac{1}{f(x)}\ .\ f'(x)$
$y = e^{f(x)}$	$y = e^{f(x)}\ .\ f'(x)$
$y = a^{f(x)}$	$y' = a^{f(x)}\ .\ f'(x)\ .\ L\,a$
$y = sin\ f(x)$	$y' = \cos f(x)\ .\ f'(x)$
$y = \cos f(x)$	$y' = -\ sin\ f(x)\ .\ f'(x)$

$y = tg\ f(x)$	$y' = \frac{1}{\cos^2 f(x)} \cdot f'(x)$
$y = cotg\ f(x)$	$y' = \frac{-1}{sin^2 f(x)} \cdot f'(x)$
$y = \text{arc}\ sin\ f(x)$	$y' = \frac{1}{\sqrt{1 - f(x)^2}} \cdot f'(x)$
$y = \text{arc}\ \cos f(x)$	$y' = \frac{-1}{\sqrt{1 - f(x)^2}} \cdot f'(x)$
$y = \text{arc}\ tg\ f(x)$	$y' = \frac{1}{1 + f(x)^2} \cdot f'(x)$
$y = \text{arc}\ cotg\ f(x)$	$y' = \frac{-1}{1 + f(x)^2} \cdot f'(x)$

EXERCICIS

1

Calculeu la derivada de les següents funcions:

1.1	$y = 5x^6 - 3x^5 + 3x^3 - 2$	$y' = 30x^5 - 15x^4 + 9x^2$
1.2	$y = x^{-4} + 2x^{-3} + x - 4$	$y' = -4x^{-5} - 6x^{-4} + 1$
1.3	$y = 3x^{10} + 2\sqrt{x} + \frac{3}{x}$	$y' = 30x^9 + \frac{1}{\sqrt{x}} - \frac{3}{x^2}$
1.4	$y = \sqrt{3} \cdot x^3 - \pi \cdot x + \sqrt{3}$	$y' = 3\sqrt{3} \cdot x^2 - \pi$
1.5	$y = 4 \sin x - 3 \cos x$	$y' = 4 \cos x + 3 \sin x$
1.6	$y = 2\sqrt{x} + \frac{2}{x} + x^5$	$y' = \frac{1}{\sqrt{x}} - \frac{2}{x^2} + 5x^4$
1.7	$y = 4x^3 + 2x^3 - x^3 + 4$	$y' = 15x^2$

1.8	$y = \frac{\pi}{2} \cdot \cos x - 3\sqrt{x}$	$y' = -\frac{\pi}{2} \sin x - \frac{3}{2\sqrt{x}}$
1.9	$y = cos(3x)$	$y' = -3\,sin(3x)$
1.10	$y = cos^2(x^3)$	$y' = -2\cos(x^3) \cdot sin(x^3) \cdot 3x^2$
1.11	$y = sin\,(3x^2-2x)$	$y' = \cos(3x^2 - 2x) \cdot (6x - 2)$
1.12	$y = cos(x^2)$	$y' = -sin(x^2) \cdot 2x$
1.13	$y = sin^3(2x^2)$	$y' = 3\,sin^2(2x^2) \cdot \cos(2x^2) \cdot 4x$
1.14	$y = cos^4(3x^4)$	$y' = 4\cos^3(3x^4) \cdot \left(-sin(3x^4)\right) \cdot 12x^3$
1.15	$y = 3\,sin^2(2x-3)$	$y' = 6\,sin(2x-3) \cdot \cos(2x-3) \cdot 2$

2

Calculeu la derivada de les següents funcions:

2.1	$y = cos^5(3x^2)$	$y' = 5\cos^4(3x^2) \cdot (-sin(3x^2)) \cdot 6x$
2.2	$y = cos\,(sinx)$	$y' = -sin(sinx) \cdot cosx$
2.3	$y = cos^2(sin(3x))$	$y' = 2\cos(sin3x) \cdot (-sin(sin3x)) \cdot \cos 3x \cdot 3$
2.4	$y = \sqrt[3]{\cos^2 x}$	$y' = \frac{-2\,cosx \cdot sinx}{3\sqrt[3]{\cos^4 x}}$
2.5	$y = \sqrt[3]{\cos^2(x^2)}$	$y' = \frac{-2\cos(x^2) \cdot sin(x^2) \cdot 2x}{3\sqrt[3]{\cos^4(x^2)}}$

2.6	$y = \sqrt{x^2 - 3x}$	$y' = \frac{2x - 3}{2\sqrt{x^2 - 3x}}$
2.7	$y = \sqrt[3]{(x^2 - 3x)^2}$	$y' = \frac{2\,(x^2 - 3x)\,(2x - 3)}{3\sqrt[3]{(x^2 - 3x)^4}}$
2.8	$y = \left(2\sqrt{x} - 3x\right)^3$	$y' = 3\left(2\sqrt{x} - 3x\right)^2 \cdot \left(\frac{1}{\sqrt{x}} - 3\right)$
2.9	$y = \sqrt[3]{\sin^2 x}$	$y' = \frac{2\,\sin x\,\cos x}{3\sqrt[3]{\sin^4 x}}$
2.10	$y = \sqrt[5]{\sin(3x)}$	$y' = \frac{3\,\cos 3x}{5\sqrt[5]{\sin^4(3x)}}$
2.11	$y = \sqrt{3x - \sin x}$	$y' = \frac{3 - \cos x}{2\sqrt{3x - \sin x}}$
2.12	$y = \left(3x^2 - \sqrt{1 - x^2}\right)^3$	$y' = 3\left(3x^2 - \sqrt{1 - x^2}\right)^2 \cdot \left(6x - \frac{-2x}{2\sqrt{1 - x^2}}\right)$
2.13	$y = \sin\left(\sqrt{3x^2 - 5x}\right)$	$y' = \cos\left(\sqrt{3x^2 - 5x}\right) \cdot \frac{6x - 5}{2\sqrt{3x^2 - 5x}}$
2.14	$y = \sqrt{\sin^3 x + (x - 1)^3}$	$y' = \frac{3\sin^2 x\,\cos x + 3(x - 1)^2}{2\sqrt{\sin^3 x + (x - 1)^3}}$

3

Calculeu la derivada de les següents funcions:

3.1 $y = \cos^3\left(x^2 - 3\sqrt{x}\right)$	$y' = -3\cos^2\left(x^2 - 3\sqrt{x}\right) \cdot \sin\left(x^2 - 3\sqrt{x}\right) \cdot \left(2x - \frac{3}{2\sqrt{x}}\right)$
3.2 $y = \frac{x}{5}$	$y' = \frac{1}{5}$

3.3 $y = \frac{5}{x}$	$y = -\frac{5}{x^2}$
3.4 $y = \frac{x^4 - 3x}{4}$	$y' = \frac{4x^3 - 3}{4}$
3.5 $y = \frac{x^3 - 3}{x}$	$y' = 2x + \frac{3}{x^2}$
3.6 $y = \frac{(x^4 - 3x)^2}{3}$	$y' = \frac{2(x^4 - 3x)^2 \cdot (4x^3 - 3)}{3}$
3.7 $y = \sqrt[3]{3x^2 - \sin x}$	$y' = \frac{6x - \cos x}{3\sqrt[3]{(3x^2 - \sin x)^2}}$
3.8 $y = ln(3x - 1)$	$y' = \frac{3}{3x - 1}$
3.9 $y = ln(x^2 - 3x)$	$y' = \frac{2x - 3}{x^2 - 3x}$
3.10 $y = ln\sqrt{x - 2}$	$y' = \frac{1}{2\sqrt{x-2}\sqrt{x-2}}$
3.11 $y = \log_2(3x^2)$	$y' = \frac{6x}{3x^2 \cdot ln2}$
3.12 $y = e^{x^2}$	$y' = e^{x^2} \cdot 2x$
3.13 $y = 2^x$	$y' = 2^x \cdot ln2$
3.14 $y = e^{x^2-2x}$	$y' = e^{x^2-2x} \cdot (2x - 2)$
3.15 $y = 3^{sen\,x}$	$y' = 3^{\sin x} \cdot \cos x \cdot ln3$
3.16 $y = 3^{sen\,x}$	$y' = \frac{3x^2}{\cos^2(x^3)}$

3.17 $y = 3\ e^{x^2-3x}$	$y' = 3\ e^{x^2-3x}\ .\ (2x-3)$
3.18 $y = \sqrt{e^{\cos x}}$	$y' = \frac{-e^{\cos x}\ \sin x}{2\sqrt{e^{\cos x}}}$
3.19 $y = 3\ tg^2 x$	$y' = \frac{6\ tgx}{\cos^2 x}$
3.20 $y = (x^2-1).(x-1)$	$y = 2x\ (x-1) + (x^2-1)$

4

Calculeu la derivada de les següents funcions:

4.1 $y = x^2\ .\ ln\ x$	$y' = 2x\ .\ ln\ x + \frac{x^2}{x}$
4.2 $y = e^{x^2}\ .\ \cos x$	$y' = e^{x^2}\ 2x\ \cos x - e^{x^2}\ \sin x$
4.3 $y = x^4\ .\ e^{3x}$	$y' = 4x^3\ .\ e^{3x} + x^4\ e^{3x}\ .\ 3$
4.4 $y = ln\ x^2\ .\ e^{\sin x}$	$y' = \frac{2x}{x^2}\ .\ e^{\sin x} + ln x^2\ e^{\sin x}\ \cos x$
4.5 $y = \frac{1}{ln\sqrt{x}}$	$y' = \frac{-\frac{1}{2\sqrt{x}}}{\sqrt{x}}{(ln\sqrt{x})^2}$
4.5. $y = ln\ x\ .\ e^{x^2-\sin x}$	$y' = \frac{1}{x}\ .\ e^{x^2-\sin x} + ln\ x\ .\ e^{x^2-\sin x}\ (2x-\cos x)$

4.6. $y = \frac{ln\, x}{3^x}$	$y' = \frac{\frac{1}{x}\, 3^x - ln\, x\, 3^x\, ln\, 3}{(3^x)^2}$
4.7. $y = ln\left(\frac{sin\, x}{e^x}\right)$	$y' = \frac{e^x}{sinx}\,\frac{\cos x\, e^x - e^x\, sin\, x}{(e^x)^2}$
4.8. $y = \sqrt{\frac{sin\, x}{x-1}}$	$y' = \frac{1}{2\sqrt{\frac{sin\, x}{x-1}}} \cdot \frac{cosx\,(x-1) - sinx}{(x-1)^2}$

5

5.Calculeu la derivada de les següents funcions:

5.1 $y = arctg\,(x^2)$	$y' = \frac{2x}{1 + x^4}$
5.2 $y = arcsin\ x^3$	$y' = \frac{3x^2}{\sqrt{1 - x^6}}$
5.3 $y = arctg\,(e^{3x})$	$y' = \frac{e^{3x} \cdot 3}{1 + (e^{3x})^2}$
5.4 $y = ln\left(\frac{tgx}{e^{3x^2}}\right)$	$y' = \frac{e^{3x^2}}{tgx} \cdot \frac{\frac{e^{3x^2}}{\cos^2 x} - tgx\, e^{3x^2}\, 6x}{\left(e^{3x^2}\right)^2}$
5.5 $y = arcsin\left(\frac{x+1}{e^x}\right)$	$y' = \frac{e^x - (x+1)\,e^x}{(e^x)^2} \Big/ \sqrt{1 - \left(\frac{x+1}{e^x}\right)^2}$

5.6 $y = ln\left(arctg(5x) \right)$	$y' = \frac{1}{arctg(5x)} \; \frac{5}{1 + (5x)^2}$
5.7 $y = arctg \sqrt{x^3}$	$y' = \frac{\frac{3x^2}{2\sqrt{x^3}}}{1 + x^3}$

6

Calculeu la derivada de les següents funcions:

6.1 $y = \ln\ (3x^4 - 3x^3)$	$y' = \frac{12x^3 - 9x^2}{3x^4 - 3x^3}$
6.2 $y = \ln\ (3x^4 - 3x^3)$	$y = -\ sin\ (x^3 - 2)\ .\ 3x^2$
6.3 $y = \sqrt{3x + 2}$	$y' = \frac{3}{2\ \sqrt{3x + 2}}$
6.4 $y = \frac{x^2 + 3}{x^3 + 3}$	$y' = \frac{2x(x^3 + 3) \ - \ 3x^2(x^2 + 3)}{(x^3 + 3)^2}$
6.5 $y = (2x^3 + 3x^2)\ .\ (3x^2 - 2x)$	$y' = (6x^2 + 6x)\ .\ (3x^2 - 2x) \ + \ (2x^3 + 3x^2)\ .\ (6x - 2)$
6.6 $y = \sqrt{x^2 + 2}$	$y' = \frac{x}{\sqrt{x^2 + 2}}$
6.7 $y = \frac{3}{x}$	$y' = -\frac{3}{x^2}$
6.8 $y = sin\left(\ \ln(3x^2 + 1)\ \right)$	$y' = \cos\left(\ \ln(3x^2 + 1)\ \right) \frac{6x}{3x^2 + 1}$
6.9 $y = \frac{4x^3 + 2}{x^2 + x}$	$y' = \frac{12x^2.(x^2 + x) \ - \ (4x^3 + 2).(2x + 1)}{(x^2 + x)^2}$

7

Calculeu la derivada de les següents funcions:

7.1 $y = \sin^2 x$	$y' = 2\sin x \cos x$
7.2 $y = \cos(x^3)$	$y' = -\sin(x^3) \cdot 3x^2$
7.3 $y = \ln\sqrt{x+1}$	$y' = \frac{1}{\sqrt{x+1}} \quad \frac{1}{2\sqrt{x+1}}$
7.4 $y = \frac{x^3 - 2}{\sin x}$	$y' = \frac{3x \sin x - (x^3 - 2)\cos x}{\sin^2 x}$
7.5 $y = \frac{x^2 - 3}{2x+5}$	$y' = \frac{2x(2x+5) - (x^2 - 3) \cdot 2}{(2x+5)^2}$
7.6 $y = \frac{3x - 2}{5}$	$y' = \frac{3}{5}$
7.7 $y = \frac{2x - 1}{x^2 - x}$	$y' = \frac{2(x^2 - x) - (2x - 1)^2}{(x^2 - x)^2}$

8

Calculeu la derivada de y en els punts indicats,

8.1 $y = 3x^2 + 2x - 1 \qquad en\ x = 1$	8
8.2 $y = \frac{1}{2x+1} \qquad en\ x = 0$	-2

8.3 $y = (x^2 - 1)^3 \qquad en\ x = \frac{1}{2}$	27/16
8.4 $y = x^2 - \cos x \qquad en\ x = \frac{\pi}{2}$	$\pi+1$
8.5 $y = \frac{x^2}{3} - \sqrt{x-1} \qquad en\ x = 2$	5/6;
8.6 $y = 2\cos(3x - \pi) \qquad en\ x = \pi$	0
8.7 $y = \frac{2}{(x-1)^2} \qquad en\ x = -1$	1/2
8.8 $y = \ln(x+1) + \sin x \qquad en\ x = 0$	2