EQUACIONS REDUÏBLES A SEGON GRAU

EXEMPLES

Ex.1

Resoleu: $4x^4-17x^2+4=0$

RAONAMENT

Efectuar canvi de variable: $t = x^2$ $\qquad 4t^2-17t+4=0$

Resoldre l'equació $t=\dfrac{17\pm\sqrt{289-64}}{8}=\dfrac{17\pm15}{8}=\begin{cases}1/4\\2\end{cases}$

Desfer el canvi de variable $\begin{cases}x^2=1/4 & x=\pm 1/2\\ x^2=2 & x=\pm\sqrt{2}\end{cases}$

Ex.2

Resoleu: $x^6-9x^3+8=0$

RAONAMENT

Efectuar canvi de variable $t = x^3$ $\qquad t^2-9t+8=0$

Resoldre l'equació $\qquad t=\dfrac{9\pm\sqrt{81-32}}{2}=\dfrac{9\pm7}{2}=\begin{cases}8\\1\end{cases}$

Desfer el canvi de variable $\begin{cases}x^3=8 & x=+\sqrt[3]{8}=2\\ x^3=1 & x=+\sqrt[3]{1}=1\end{cases}$

EXERCICIS

1

Equacions biquadrades

Resoleu:

1. $x^4-5x^2+4=0$	Sol: $x=\pm 1, x=\pm 2$
2. $x^4+2x^2-3=0$	Sol: $x=\pm 1$
3. $x^6-9x^3+8=0$	Sol: $x=2, x=1$
4. $x^6-26x^3-27=0$	Sol: $x=-1, x=3$
5. $6x^4+2x^2-8=0$	Sol: $x=\pm 1$

RAONAMENT

$6x^4+2x^2-8=0 \quad \rightarrow \quad 3\,(x^2)^2 + 1\,(x^2) - 4 = 0$

$$x^2 = \frac{-1 \pm \sqrt{1+48}}{6} = \begin{cases} 1 \\ -4/3 \end{cases} \quad \rightarrow \quad x = \pm 1$$

6. $x^4-4x^2=0$	Sol: $x=0, x=\pm 2$
7. $4x^4-17x^2+4=0$	Sol: $x=\pm 2, x=\pm 1/2$
8. $9x^4-3x^2+4=0$	Sol: $x=\pm 1/3, x=\pm 2$
9. $x^4-6x^2-27=0$	Sol: $x=\pm 3$
10. $x^6+7x^3-8=0$	Sol: $x=1, x=-2$

RAONAMENT

$x^6+7x^3-8=0 \quad \rightarrow \quad (x^3)^2 + 7\,(x^3) - 8 = 0$

$$x^3 = \frac{-7 \pm \sqrt{49+32}}{2} = \begin{cases} 1 \\ -8 \end{cases} \quad \rightarrow \quad \begin{cases} x = \sqrt[3]{1} = 1 \\ x = \sqrt[3]{-8} = -2 \end{cases}$$

11. $x^4-2x^2-8=0$	Sol: $x=\pm 2$

12. $x^6+28x^3+27=0$	Sol: $x=-1$, $x=-3$
13. $x^4-7x^2+12=0$	Sol: $x=\pm\sqrt{3}$; $x=\pm 2$
14. $x^4-9x^2+18=0$	Sol: $x=\pm\sqrt{3}$; $x=\pm\sqrt{6}$
15. $x^4-5x^2+6=0$	Sol: $x=\pm\sqrt{2}$; $x=\pm\sqrt{3}$
RAONAMENT $x^4-5x^2+6=0 \rightarrow (x^2)^2-5(x^2)+6=0$ $x^2=\frac{5\pm\sqrt{25-24}}{2}=\begin{cases}3\\2\end{cases} \rightarrow x=\begin{cases}\pm\sqrt{3}\\\pm\sqrt{2}\end{cases}$	
16. $x^4+8x^2+15=0$	Sol: no té solució
17. $x^4-6x^2-27=0$	Sol: $x=\pm 3$
18. $x^4-6x^2+9=0$	Sol: $x=\pm\sqrt{3}$
19. $x^4+6x^2=-9$	Sol: no té solució
20. $x^4-9x^2+14=0$	Sol: $x=\pm\sqrt{2}$; $x=\pm\sqrt{7}$
RAONAMENT $x^4-9x^2+14=0 \rightarrow (x^2)^2-9(x^2)+14=0$ $x^2=\frac{9\pm\sqrt{81-56}}{2}=\begin{cases}7\\2\end{cases} \rightarrow x=\begin{cases}\pm\sqrt{7}\\\pm\sqrt{2}\end{cases}$	
21. $x^4-6x^2+8=0$	Sol: $x=\pm 2$; $x=\pm\sqrt{2}$
22. $2x^4+10x^2-48=0$	Sol: $x=\pm\sqrt{3}$

23. $x^4-x^2=20$	Sol: ; $x=\pm\sqrt{5}$
24. $x^4=5x^2+6$	Sol: $x=\pm\sqrt{6}$
25. $2x^4-5x^2+3=0$	Sol: $x=\pm 1$; $x=\pm\sqrt{\frac{3}{2}}$

RAONAMENT

$2x^4-5x^2+3=0 \quad \rightarrow \quad 2\,(x^2)^2-5\,(x^2)+3=0$

$$x^2=\frac{5\pm\sqrt{25-24}}{4}=\begin{cases}3/2\\1\end{cases} \quad \rightarrow \quad x=\begin{cases}\pm\sqrt{3/2}\\\pm 1\end{cases}$$

26. $x^6+10x^3+25=0$	Sol: $x=-\sqrt[3]{5}$
27. $x^{10}+9=10x^5$	Sol: $x=1$; $x=\sqrt[5]{9}$
28. $3x^6-39x^3+108=0$	Sol: $x=\sqrt[3]{4}$; $x=\sqrt[3]{9}$
29. $2x^8-9x^4+9=0$	Sol: $x=\pm\sqrt[4]{3}$; $x=\pm\sqrt[4]{\frac{3}{2}}$
30. $3x^{14}+2x^7=8$	Sol: $x=-\sqrt[7]{2}$; $x=\sqrt[7]{\frac{4}{3}}$

RAONAMENT

$3x^{14}+2x^7=8 \quad \rightarrow \quad 3\,(x^7)^2+2\,(x^7)-8=0$

$$x^7=\frac{-2\pm\sqrt{4+96}}{6}=\begin{cases}4/3\\-2\end{cases} \quad \rightarrow \quad x=\begin{cases}\sqrt[7]{4/3}\\\sqrt[7]{-2}=-\sqrt[7]{2}\end{cases}$$