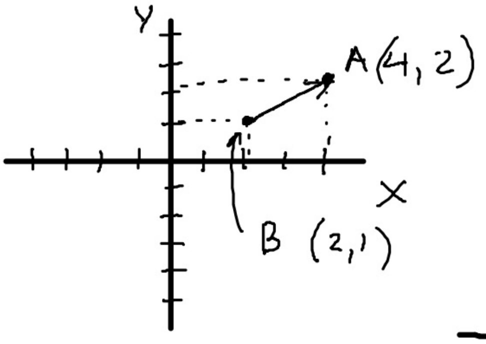
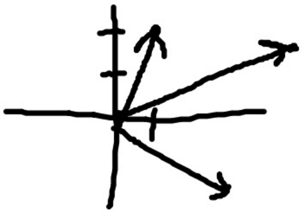
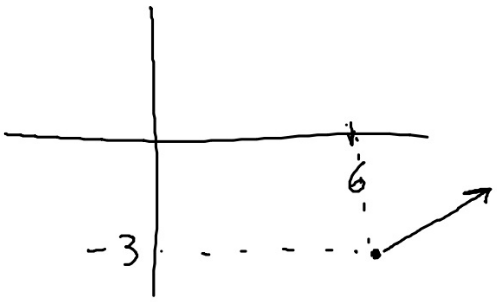
GEOMETRIA



VECTOR $\overrightarrow{BA} = A - B =$

$= (4,2) - (2,1) = \overrightarrow{(2,1)}$





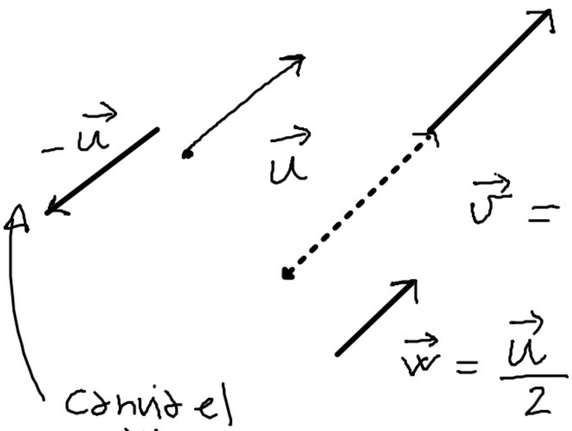
$P = (6,-3)$

$P + \overrightarrow{BA} = (6,-3) + \overrightarrow{(2,1)} =$

$= \overrightarrow{(8,-2)}$

Operacions amb vectors

* nombre · vector = vector



$\vec{v} = 2 \cdot \vec{u}$

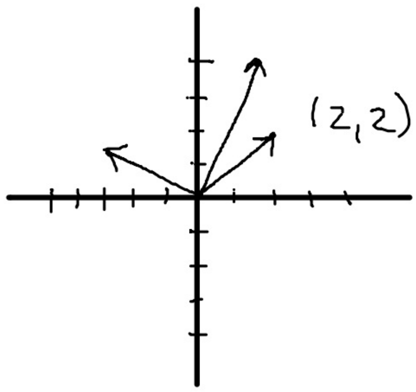
$\vec{w} = \dfrac{\vec{u}}{2}$

canvia el sentit

En general, si multipliquem un vector per un nombre K,

si $K > 1$ augmentem $K \cdot \vec{v}$

$K < 1$ disminuim

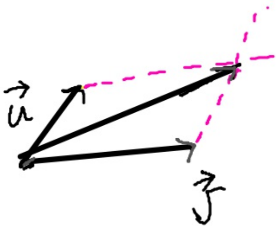


$(0,0)$ no marca cap direcció

* vector ± vector = vector

$\vec{u} = (2,5)$

$\vec{v} = (3,7)$



$\vec{u} + \vec{v} = (5,12)$

Exercicis

a) Troba els components del vector $\overrightarrow{AB}$ on $A=(2,1)$

$B=(5,7)$

$$B-A = (5-2, 7-1) = (3, 6)$$

b) Calcula x i y perquè els vectors $\vec{u}$ i $\vec{v}$ siguin iguals:

$\vec{u} = (x, 5)$
$\vec{v} = (6, y)$
$x=6$
$y=5$

$\vec{u} = (3x, 8)$
$\vec{v} = (2, 4y)$
$3x = 2 \Rightarrow x = 2/3$
$8 = 4y \Rightarrow y = 2$

$\vec{u} = (x-1, 3)$
$\vec{v} = (2x, y)$
$x-1 = 2x \Rightarrow \boxed{-1 = x}$

c) Si $\vec{u} = \overrightarrow{(2,-1)}$, $\vec{v} = \overrightarrow{(5,3)}$ calcula $3\vec{u}-2\vec{v} = \overrightarrow{(-4,-9)}$

$\vec{v}+2\vec{u} = \overrightarrow{(9,1)}$

UNA COMBINACIÓ LINEAL de VECTORS

$$3\vec{u}-2\vec{v} = (6,-3)-(10,6) = \overrightarrow{(-4,-9)}$$

$$\vec{v}+2\vec{u} = (5,3)+(4,-2) = \overrightarrow{(9,1)}$$

$K \cdot \vec{u} =$ vector

$\vec{u} \pm \vec{v} =$ vector

$\vec{u} \cdot \vec{v} =$ nombre o escolar, no un vector

?

PRODUCTE ESCALAR de VECTORS

DEFINICIÓ

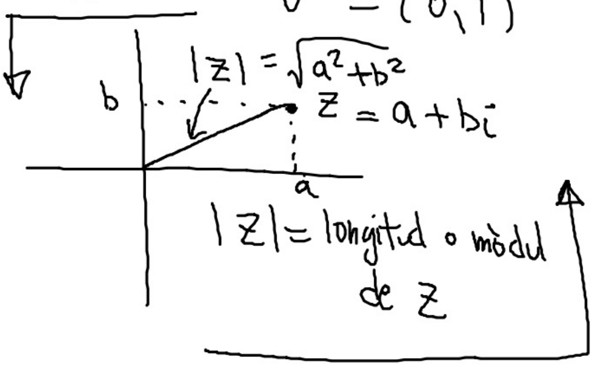
$\vec{u} = (\overrightarrow{a, b})$ $\quad \vec{u} \cdot \vec{v} = (a,b) \cdot (c,d) = a \cdot c + b \cdot d$

$\vec{v} = (\overrightarrow{c, d})$ $\quad \langle \vec{u}, \vec{v} \rangle$

vectors

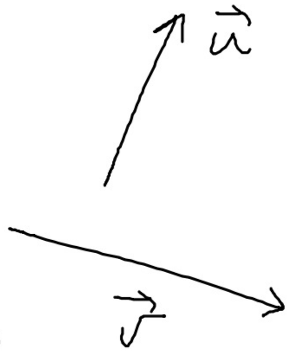
Exemple $\quad \vec{u} = (\overrightarrow{1, 0})$ $\quad$ Calcula $\vec{u} \cdot \vec{v} = (1,0) \cdot (0,1) = 0 + 0 = 0$

$\vec{v} = (\overrightarrow{0, 1})$



Longitud o mòdul de $\vec{u} = (\overrightarrow{a, b})$

$$|\vec{u}| = \sqrt{a^2 + b^2}$$

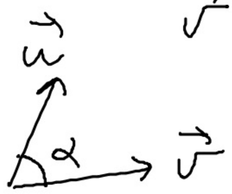


Són perpendiculars $\perp$

Són paral.lels $\parallel$

$\vec{u} = \overrightarrow{(a,b)}$

$\vec{v} = \overrightarrow{(c,d)}$



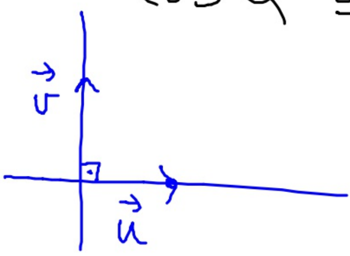
EXEMPLE $\vec{u} = \overrightarrow{(1,0)}$

$\vec{v} = \overrightarrow{(0,1)}$

Definim $\cos\alpha = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \dfrac{(1,0)\cdot(0,1)}{1\cdot 1} = \dfrac{0+0}{1} =$

$= \dfrac{0}{1} = 0 \quad \cos\alpha = 0$

$\boxed{\alpha = 90^\circ}$



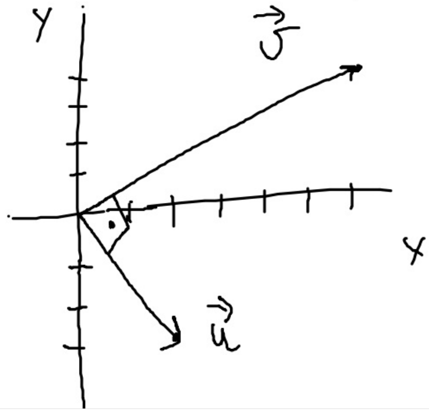
Exercicis

ⓐ Calcula $\vec{u}\cdot\vec{v}$, on $\vec{u}=(2,-3)$, $\vec{v}=(6,4)$. Quin angle formen els vectors?

$$\vec{u}\cdot\vec{v}=(2,-3)\cdot(6,4)=12-12=0$$

$$\cos\alpha=\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}=\frac{0}{\sqrt{13}\cdot\sqrt{52}}=0$$

$$\alpha=\arccos 0=90^{\circ}$$

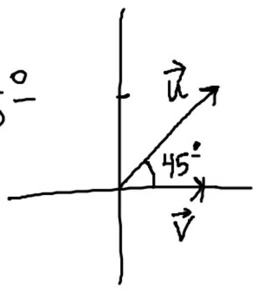


i si $\vec{u}=(1,1)$, $\vec{v}=(1,0)$?

$$\vec{u}\cdot\vec{v}=(1,1)\cdot(1,0)=1+0=1$$

$$\cos\alpha=\frac{1}{\sqrt{2}\cdot 1}=\frac{\sqrt{2}}{2}$$

$$\alpha=\arccos\frac{\sqrt{2}}{2}=45^{\circ}$$



Si α és l'angle que formen u i v: $\cos\alpha = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$

EXERCICIS PROVES ACCÉS VECTOR

1.

Donats els vectors: $\vec{u} = (-3,4)$ i $\vec{v} = (5,2)$ calculeu:

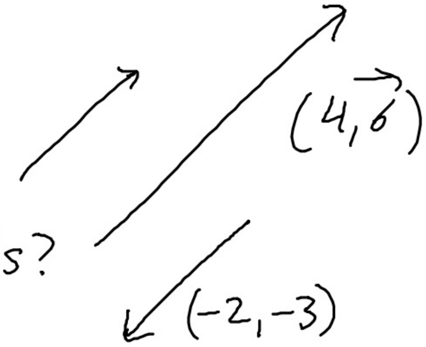
a) $3\vec{u} - 2\vec{v}$. $= 3\cdot(-3,4) - 2\cdot(5,2) = (-9,12) - (10,4) = (-19,8)$

b) El mòdul del vector $\vec{u}$. $|\vec{u}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

c) El producte escalar $\vec{u}\cdot\vec{v}$. $= (-3,4)\cdot(5,2) = -15+8 = -7$

d) L'angle que formen els vectors $\vec{u}$ i $\vec{v}$ $\cos\alpha = \dfrac{u\cdot v}{|u|\cdot|v|} = \dfrac{-7}{5\sqrt{29}} = -0{,}26 \Rightarrow \alpha = 105{,}07°$

$\vec{u} = \overrightarrow{(2,3)}$



* Com detectar vectors paral.lels?

$\vec{u} = (4, 7)$

$\vec{v} = (24, 42)$

$$\frac{24}{4} \stackrel{?}{=} \frac{42}{7}$$

$$6 = 6 \checkmark$$

TRUC

$\vec{u} = (a, b)$

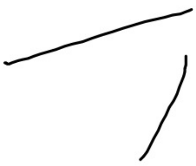
$\vec{v} = (c, d)$

$\frac{a}{c} = \frac{b}{d}$ són $\parallel$

$\frac{a}{c} \neq \frac{b}{d}$ no són $\parallel$

$\parallel$

paral.lels



Com sabem si $\vec{u}, \vec{v}$ són perpendiculars? = ORTOGONAL

Fem $\vec{u} \cdot \vec{v}$, si surt 0 són perpendiculars

EX

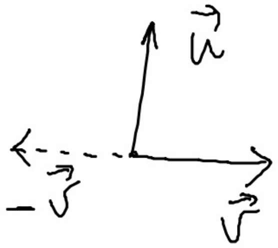
(a) $\vec{u} = (2,3)$

$\vec{v} = (-3,2)$

$\vec{u} \cdot \vec{v} = -6 + 6 = 0$ són $\perp$ ✓

(b) $\vec{u} = (5,-4)$

$\vec{v} = \begin{cases} (4,5) \\ (-4,-5) \end{cases}$



Calcula un vector paral.lel a (4,3) de mòdul 10. I de mòdul 1?

$(4,3)$ $\quad |\vec{u}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

Vector paral.lel: $\quad 2\cdot(4,3) = (8,6)$ ó $(-8,-6)$

NO CAL $\Rightarrow$ $x\cdot(4,3) = (4x, 3x)$ $\quad \sqrt{(4x)^2+(3x)^2} = 10$

$(4x)^2 + (3x)^2 = 10^2 \Rightarrow 16x^2 + 9x^2 = 100 \Rightarrow 25x^2 = 100 \Rightarrow x^2 = \frac{100}{25} = 4$

$x^2 = 4 \Rightarrow x = \sqrt{4} = \pm 2$ $\quad +2\cdot(4,3) = (8,6)$ ó $-2\cdot(4,3) = (-8,-6)$

Mòdul 1

$\dots \quad 25x^2 = 1 \Rightarrow x^2 = \frac{1}{25} \Rightarrow x = \sqrt{\frac{1}{25}} = \pm\frac{1}{5} \Rightarrow \left(\frac{4}{5}, \frac{3}{5}\right)$ $\left(-\frac{4}{5}, \frac{-3}{5}\right)$

Dados $\vec{u}(2, 3)$, $\vec{v}(-3, 1)$ y $\vec{w}(5, 2)$, calcula:

a) $(3\vec{u} + 2\vec{v})\vec{w}$

ALTRA FORMA

b) $\vec{u} \cdot \vec{w} - \vec{v} \cdot \vec{w}$ $= (\vec{u} - \vec{v}) \cdot \vec{w} = (5, 2) \cdot (5, 2) = 29$

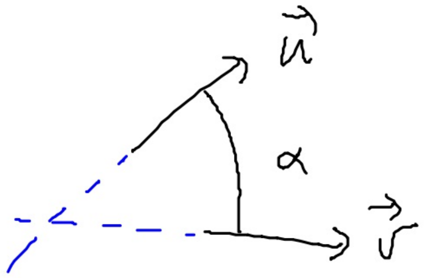
a) $3\vec{u} + 2\vec{v} = 3(2, 3) + 2(-3, 1) = (6, 9) + (-6, 2) = (0, 11)$

$(3\vec{u} + 2\vec{v}) \cdot \vec{w} = (0, 11) \cdot (5, 2) = 0 \cdot 5 + 11 \cdot 2 = 0 + 22 = 22$

b) $\left.\begin{array}{l} \vec{u} \cdot \vec{w} = (2, 3) \cdot (5, 2) = 10 + 6 = 16 \\ \vec{v} \cdot \vec{w} = (-3, 1) \cdot (5, 2) = -15 + 2 = -13 \end{array}\right\} \rightarrow$

$\rightarrow \vec{u} \cdot \vec{w} - \vec{v} \cdot \vec{w} = 16 - (-13) = 16 + 13 = 29$

Si $|\vec{u}| = 3$, $|\vec{v}| = 5$ y $\vec{u} \cdot \vec{v} = -2$, averigua el ángulo $(\vec{u}, \vec{v})$. (Usa la calculadora).



$$\underset{\alpha}{\cos\,(\widehat{\vec{u}, \vec{v}})} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|} = \frac{-2}{3 \cdot 5} = -\frac{2}{15} \;\rightarrow\; \underset{\alpha}{(\widehat{\vec{u}, \vec{v}})} = 97° \, 39' \, 44'' \quad 97{,}66^{o}$$