Factoritza:

$$P(x) = x^3 + 9x^2 - x - 9$$

$$\mathrm{Div}(-9) = \{\pm 1, \pm 3, \pm 9\}$$

$$P(1) = 1 + 9 - 1 - 9 = 0 \checkmark \qquad \boxed{x = 1}$$

$$\begin{array}{r|rrrr}
 & 1 & 9 & -1 & -9 \\
1 & & 1 & 10 & 9 \\
\hline
 & 1 & 10 & 9 & \boxed{0} \\
-1 & & -1 & -9 & \\
\hline
 & 1 & 9 & \boxed{0} &
\end{array}$$

$$\underbrace{1 \quad 9}_{x+9}$$

$$P(x) = (x-1)\underbrace{(x^2 + 10x + 9)}_{Q(x)}$$

$$Q(-1) = 0 \checkmark$$

$$= (x-1) \cdot (x+1)(x+9)$$

$$\boxed{\text{ARRELS o ZEROS són } x = 1,\ x = -1 \text{ i } x = -9}$$

$$x^4+3x^3-6x^2+7x=$$

$$= x\cdot\underbrace{(x^3+3x^2-6x+7)}_{\text{les altres 3 arrels estan aquí}}$$

$$\boxed{x=0}$$

x^3+3x^2-6x+7 $\quad$ $Div(7)=\{\pm 1, \pm 7\}$

	1	3	−6	7
7		7	70	448
	1	10	64	455

	1	3	−6	7
−7		−7	28	−154
	1	−4	22	−147

No puc trobar les arrels reals del polinomi

Donat $P(x) = x^4 + 3x^3 + x^2 + Kx - 2$,

troba el valor de K perquè la divisió per $x+1$:

(a) Sigui exacta

(b) Tingui per residu -3

(a)

	1	3	1	K	-2
-1		-1	-2	1	$-K-1$
	1	2	-1	$K+1$	$-K-3$

$$-K-3 = 0 \Rightarrow \boxed{-3 = K}$$

(b)

$$\text{Residu} = -K-3 = -3 \Rightarrow -K = 3-3$$

$$-K = 0 \Rightarrow \boxed{K=0}$$

Dedueix el valor de K perquè la divisió de

$$P(x) = 2x^4 + 11x^3 + 3x^2 - 11x + K$$ per

$x+5$ sigui exacta.

Quin valor ha de prendre K perquè el residu d'aquesta divisió sigui 15?

$$\begin{array}{r|rrrrr}
 & 2 & 11 & 3 & -11 & K \\
-5 & & -10 & -5 & 10 & 5 \\
\hline
 & 2 & 1 & -2 & -1 & \boxed{K+5}
\end{array}$$

$$R = K+5 = 0 \Rightarrow \boxed{K=-5}$$

$$R = K+5 = 15 \Rightarrow \boxed{K=10}$$

FRACCIONS ALGÈBRIQUES

$\frac{2}{3}$ fracció numèrica o nombre racional

$\frac{P(x)}{Q(x)} = \frac{2x-4}{3x^2+6x-7}$ fracció algèbrica

Valor numèric de $\frac{P(x)}{Q(x)}$ si $x=0$

$$\frac{P(0)}{Q(0)} = \frac{-4}{-7}$$

$$\frac{4}{6} = \frac{\cancel{2}\cdot 2}{\cancel{2}\cdot 3}$$

$$\frac{7x^2-x}{3x} = \frac{\cancel{x}\cdot(7x-1)}{3\cdot\cancel{x}} = \frac{7x-1}{3}$$

De vegades al operar una mica per simplificar

$$\frac{x^2-4x+4}{x^2-4} = \frac{(x-2)^2}{(x+2)\cdot(x-2)} = \frac{(x-2)\cdot\cancel{(x-2)}}{(x+2)\cdot\cancel{(x-2)}} = \frac{x-2}{x+2}$$

$$\frac{1}{2}+\frac{2}{3} = \frac{3+2\cdot 2}{6} = \frac{7}{6}$$

$$2-x^2=0$$

$$\Rightarrow 2=x^2 \Rightarrow \pm\sqrt{2}=x$$

$$\frac{1}{x+1}-\frac{x}{x+2} = \frac{x+2-x\cdot(x+1)}{(x+1)(x+2)} = \frac{\cancel{x}+2-x^2-\cancel{x}}{(x+1)(x+2)} = \frac{2-x^2}{(x+1)(x+2)}$$

com que no hi ha cap factor comú dalt i baix, ja no puc simplificar

Simplifica (a) $\frac{4x}{x+1} + \frac{3x}{x-1} = \frac{4x(x-1) + 3x(x+1)}{(x+1)(x-1)} =$

$$= \frac{4x^2 - 4x + 3x^2 + 3x}{x^2 - 1} = \boxed{\frac{7x^2 - x}{x^2 - 1}}$$

EQUACIONS

$$2x + 6 = -9 \Rightarrow 2x = -15 \quad \boxed{x = -\frac{15}{2}}$$

$$3x^2 - 5x + 6 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 4 \cdot 3 \cdot 6}}{2 \cdot 3} = \dots$$

– EQ. BIQUADRADES

ⓐ $\underset{(x^2)^2}{x^4} - 29x^2 + 100 = 0 \quad \boxed{z = x^2} \quad z^2 - 29z + 100 = 0$

$$z = \frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2a} = \frac{29 \pm \sqrt{(-29)^2 - 4 \cdot 1 \cdot 100}}{2 \cdot 1} = \frac{29 \pm \sqrt{441}}{2} = \frac{29 \pm 21}{2} \begin{cases} 25 \\ 4 \end{cases}$$

$$z = 25 = x^2 \Rightarrow x = \sqrt{25} = \boxed{\pm 5}$$

$$z = 4 = x^2 \Rightarrow x = \sqrt{4} = \boxed{\pm 2}$$

Per tant, $x = 5, -5, 2, -2$ són les quatre solucions

b) $x^4 - 8x^2 - 9 = 0$ $\boxed{z = x^2}$

$$z^2 - 8z - 9 = 0 \Rightarrow z = \frac{8 \pm \sqrt{64 - 4 \cdot 1 \cdot (-9)}}{2 \cdot 1} = \frac{8 \pm 10}{2} \begin{cases} 9 \\ -1 \end{cases}$$

$$z = 9 = x^2 \Rightarrow x = \sqrt{9} = \boxed{\pm 3}$$

$$z = -1 = x^2 \Rightarrow x = \sqrt{-1}$$ són imaginàries

$\boxed{(x+3) \cdot (x-3)(x^2+1)}$

FACTORITZACIÓ

– EQUACIONS IRRACIONALS

$$2x + \sqrt{1+x} = 3x - 5$$

com apareix una arrel, es tracta d'una eq. irracional

$$\sqrt{1+x} = 3x - 5 - 2x$$

$$\sqrt{1+x} = x - 5 \Rightarrow \left(\cancel{\sqrt{1+x}}\right)^{\cancel{2}} = (x-5)^2$$

elevem tota l'equació al quadrat

$$1 + x = x^2 + 25 - 10x$$

$$0 = x^2 - 11x + 24$$ eq. de 2n. grau, si la resolem dóna $x = 8$, $x = 3$

! COMPROVEM si són o no SOLUCIONS

$2\cdot 8 + \sqrt{1+8} \stackrel{?}{=} 3\cdot 8 - 5 \Rightarrow 16 + 3 = 24 - 5$ ✓

$2\cdot 3 + \sqrt{1+3} \stackrel{?}{=} 3\cdot 3 - 5 \Rightarrow 6 + 2 \neq 4$ ✗

$\boxed{\text{Només és } x = 8}$

$(a \pm b)^2 = a^2 + b^2 \pm 2ab$

ⓐ $\sqrt{2x-6} - 4 = 0$

$\sqrt{2x-6} = 4 \Rightarrow \left(\sqrt{2x-6}\right)^2 = 4^2$

COMPROVEM

$\sqrt{2 \cdot 11 - 6} - 4 \overset{?}{=} 0$

$\sqrt{16} - 4 = 0$ ✓

$2x - 6 = 16 \Rightarrow 2x = 22$

$\boxed{x = \frac{22}{2} = 11}$

ⓑ $\sqrt{11-2x} - x = 2 \Rightarrow \left(\sqrt{11-2x}\right)^2 = (2+x)^2$

$11 - 2x = 4 + x^2 + 4x \Rightarrow \boxed{x^2 + 6x - 7 = 0}$

$x = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-7)}}{2} = \frac{-6 \pm \sqrt{36+28}}{2} = \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2} \begin{cases} 1 \\ -7 \end{cases}$

COMPROVEM

$\boxed{x=1}$ $\sqrt{11 - 2 \cdot 1} - 1 \overset{?}{=} 2$ $\quad 3 - 1 = 2$ ✓ $\quad \boxed{x = -7}$ $\sqrt{11 - 2 \cdot (-7)} + 7 \overset{?}{=} 2$ $\quad 5 + 7 \neq 2$ No és solució

SISTEMES D'EQUACIONS.

$$\left.\begin{array}{l} x + 2y = 5 \\ 2x - y = 0 \end{array}\right\}$$

ⓐ SUBSTITUCIÓ

$$\boxed{2x = y}$$

He aïllat l'y

$$x + 2\cdot(2x) = 5$$

$$x + 4x = 5 \Rightarrow 5x = 5$$

$$\boxed{x = \frac{5}{5} = 1}$$

$$\boxed{y = 2\cdot 1 = 2}$$

COMPROVACIÓ

$1 + 2\cdot 2 = 5$ ✓

$2\cdot 1 - 2 = 0$ ✓

ⓑ REDUCCIÓ

$$x + 2y = 5$$

$$2\cdot(2x - y = 0)$$

$$5x \quad \cancel{} = 5 \Rightarrow \boxed{x = \frac{5}{5} = 1}$$

Substituïm:

$$1 + 2y = 5 \Rightarrow 2y = 4 \Rightarrow \boxed{y = \frac{4}{2} = 2}$$

~~EXERCICI~~

$$\left.\begin{array}{l} x - y = 8 \\ 5x + 8y = 14 \end{array}\right\}$$

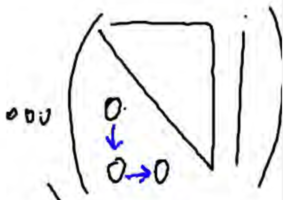
REDUCCIÓ

$$\left.\begin{array}{l} x-y=8\\ 5x+8y=14\end{array}\right\} \qquad \begin{array}{r} -5\,(x-y=8)\\ 5x+8y=14\\ \hline 13y=-26 \end{array} \Rightarrow \boxed{y=\frac{-26}{13}=-2}$$

$$x-(-2)=8 \Rightarrow x+2=8 \Rightarrow \boxed{x=6}$$

MÈTODE de GAUSS

$$\left.\begin{array}{l} x+y+z=9\\ 2x-y+3z=14\\ 2x+y-z=0\end{array}\right\} \quad \begin{array}{c} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9\\ 2 & -1 & 3 & 14\\ 2 & 1 & -1 & 0\end{array}\right) \end{array}\begin{array}{l} \to F_1\\ \to F_2\\ \to F_3\end{array}$$



$$\xrightarrow{\boxed{\begin{array}{l}-2F_1+F_2\\ -2F_1+F_3\end{array}}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9\\ 0 & -3 & 1 & -4\\ 0 & -1 & -3 & -18\end{array}\right) \xrightarrow{\boxed{F_2 \leftrightarrow F_3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9\\ 0 & -1 & -3 & -18\\ 0 & -3 & 1 & -4\end{array}\right) \xrightarrow{\boxed{-3F_2+F_3}}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -1 & -3 & -18 \\ 0 & 0 & 10 & 50 \end{array}\right) \Rightarrow$$

$$10z = 50 \Rightarrow \boxed{z = \frac{50}{10} = 5}$$

$$-y - 3\cdot \overset{z}{5} = -18$$

$$-y = 15 - 18 = -3$$

$$\boxed{y = 3}$$

$$x + y + z = 9$$

$$x + 3 + 5 = 9 \Rightarrow \boxed{x = 1}$$

$$\boxed{\begin{array}{ll} \text{Solució} & x = 1 \\ \text{única} & y = 3 \\ & z = 5 \end{array}}$$

EXERCICI

$$\left.\begin{array}{r} x - y + z = 2 \\ x + y + 2z = 9 \\ 2x + 2y + 4z = 15 \end{array}\right\}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 1 & 1 & 2 & 9 \\ 2 & 2 & 4 & 15 \end{array}\right) \underset{\boxed{\begin{array}{c} -F_1+F_2 \\ -2F_1+F_3 \end{array}}}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 7 \\ 0 & 4 & 2 & 11 \end{array}\right) \underset{\boxed{-2F_2+F_3}}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 7 \\ 0 & 0 & 0 & -3 \end{array}\right) \iff \begin{array}{l} 0 \cdot z = -3 \\ 0 = -3 \end{array}$$

! IMPOSSIBLE

Per tant el sistema no té solució, és un sistema incompatible

EXERCICIS

$$\begin{cases} x + y = 1 \\ \phantom{x+{}} y + z = -2 \\ x + z = 3 \end{cases}$$

$$\left.\begin{array}{l} x + y = 1 \\ \phantom{x+{}} y + z = -2 \\ x + z = 3 \end{array}\right\} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -2 \\ 1 & 0 & 1 & 3 \end{array}\right) \xrightarrow{\begin{array}{l} (1.^a) \\ (2.^a) \\ (3.^a) - (1.^a) \end{array}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & -1 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{\begin{array}{l} (1.^a) \\ (2.^a) \\ (3.^a) + (2.^a) \end{array}} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 2 & 0 \end{array}\right) \to \left.\begin{array}{r} x + y = 1 \\ y + z = -2 \\ 2z = 0 \end{array}\right\}$$

$z = 0 \qquad y = -2 - z = -2 \qquad x = 1 - y = 3$

Solución: $(3, -2, 0)$

$$\begin{cases} 3x + 4y - z = 3 \\ 6x - 6y + 2z = -16 \\ x - y + 2z = -6 \end{cases}$$

$$\left.\begin{array}{r} 3x + 4y - z = 3 \\ 6x - 6y + 2z = -16 \\ x - y + 2z = -6 \end{array}\right\} \left(\begin{array}{ccc|c} 3 & 4 & -1 & 3 \\ 6 & -6 & 2 & -16 \\ 1 & -1 & 2 & -6 \end{array}\right) \to \begin{array}{l} (3.^a) \\ (2.^a) : 2 \\ (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 3 & -3 & 1 & -8 \\ 3 & 4 & -1 & 3 \end{array}\right)$$

$$\to \begin{array}{l} (1.^a) \\ (2.^a) - 3 \cdot (1.^a) \\ (3.^a) - 3 \cdot (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 0 & 0 & -5 & 10 \\ 0 & 7 & -7 & 21 \end{array}\right) \to \begin{array}{l} (1.^a) \\ (2.^a) : (-5) \\ (3.^a) : 7 \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 2 & -6 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & -1 & 3 \end{array}\right)$$

$$\to \left.\begin{array}{r} x - y + 2z = -6 \\ z = -2 \\ y - z = 3 \end{array}\right\} \begin{array}{l} y = 3 + z = 3 - 2 = 1 \\ x = -6 + y - 2z = -6 + 1 + 4 = -1 \end{array}$$

Solución: $(-1, 1, -2)$

$$\begin{cases} 2x + 5y = 16 \\ x + 3y - 2z = -2 \\ x + z = 4 \end{cases}$$

$$\left.\begin{array}{r} 2x + 5y = 16 \\ x + 3y - 2z = -2 \\ x + z = 4 \end{array}\right\} \left(\begin{array}{ccc|c} 2 & 5 & 0 & 16 \\ 1 & 3 & -2 & -2 \\ 1 & 0 & 1 & 4 \end{array}\right) \to \begin{array}{l} (1.^a) \\ (2.^a) + 2 \cdot (3.^a) \\ (3.^a) \end{array} \left(\begin{array}{ccc|c} 2 & 5 & 0 & 16 \\ 3 & 3 & 0 & 6 \\ 1 & 0 & 1 & 4 \end{array}\right)$$

$$\to \begin{array}{l} (1.^a) \\ (2.^a) : 3 \\ (3.^a) \end{array} \left(\begin{array}{ccc|c} 2 & 5 & 0 & 16 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 4 \end{array}\right) \to \begin{array}{l} (1.^a) - 5 \cdot (2.^a) \\ (2.^a) \\ (3.^a) \end{array} \left(\begin{array}{ccc|c} -3 & 0 & 0 & 6 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 4 \end{array}\right) \to$$

$$\to \left.\begin{array}{r} -3x = 6 \\ x + y = 2 \\ x + z = 4 \end{array}\right\} \begin{array}{l} x = -2 \\ y = 2 - x = 4 \\ z = 4 - x = 6 \end{array}$$

Solución: $(-2, 4, 6)$

$$\begin{cases} -x + 2y - z = 1 \\ 2x - 4y + 2z = 3 \\ x + y + z = 2 \end{cases}$$

$$\left.\begin{array}{r} -x + 2y - z = 1 \\ 2x - 4y + 2z = 3 \\ x + y + z = 2 \end{array}\right\} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 1 \\ 2 & -4 & 2 & 3 \\ 1 & 1 & 1 & 2 \end{array}\right) \to \begin{array}{l} (3.^a) \\ (2.^a) \\ (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & -4 & 2 & 3 \\ -1 & 2 & -1 & 1 \end{array}\right) \to$$

$$\to \begin{array}{l} (1.^a) \\ (2.^a) - 2 \cdot (1.^a) \\ (3.^a) + (1.^a) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -6 & 0 & -1 \\ 0 & 3 & 0 & 3 \end{array}\right) \to \begin{array}{l} (1.^a) \\ (2.^a) + 2 \cdot (3.^a) \\ (3.^a) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 3 & 0 & 3 \end{array}\right)$$

La segunda ecuación es imposible: $0x + 0y + 0z = 5$

El sistema es *incompatible*.

$$\begin{cases} -x + y + 3z = -2 \\ 4x + 2y - z = 5 \\ 2x + 4y - 7z = 1 \end{cases}$$

$$\left.\begin{array}{r} -x + y + 3z = -2 \\ 4x + 2y - z = 5 \\ 2x + 4y - 7z = 1 \end{array}\right\} \left(\begin{array}{ccc|c} -1 & 1 & 3 & -2 \\ 4 & 2 & -1 & 5 \\ 2 & 4 & -7 & 1 \end{array}\right) \to \begin{array}{l} (1.^a) \\ (2.^a) + 4 \cdot (1.^a) \\ (3.^a) + 2 \cdot (1.^a) \end{array} \left(\begin{array}{ccc|c} -1 & 1 & 3 & -2 \\ 0 & 6 & 11 & -3 \\ 0 & 6 & -1 & -3 \end{array}\right)$$

$$\to \begin{array}{l} (1.^a) \\ (2.^a) \\ (3.^a) - (1.^a) \end{array} \left(\begin{array}{ccc|c} -1 & 1 & 3 & -2 \\ 0 & 6 & 11 & -3 \\ 0 & 0 & -12 & 0 \end{array}\right) \to \text{ Sistema } \textit{compatible determinado.}$$

Lo resolvemos:

$$\left.\begin{array}{r} -x + y + 3z = -2 \\ 6y + 11z = -3 \\ z = 0 \end{array}\right\} \quad y = -\frac{1}{2} \quad x = y + 3z + 2 = \frac{3}{2}$$

Solución: $\left(\frac{3}{2}, -\frac{1}{2}, 0\right)$