(b) $a = 2 \cdot 10^3$

$b = 0{,}3 \cdot 10^{-2}$

Calcula $a \cdot b$ i escriu-lo en NOTACIÓ CIENTÍFICA.

$$a \cdot b = \underbrace{2 \cdot 10^3}_{a} \cdot \underbrace{0{,}3 \cdot 10^{-2}}_{b} = 0{,}6 \cdot 10^1 = \underbrace{6 \cdot 10^{-1}}_{0{,}6} \cdot 10^1$$

$$= 6 \cdot 10^0 = 6 \qquad (10^0 = 1)$$

(c) $a = 200 \cdot 10^3$

$b = 4 \cdot 10^{-5}$

$$\frac{a}{b} = \frac{200 \cdot 10^3}{4 \cdot 10^{-5}} = 50 \cdot 10^{3-(-5)} = 50 \cdot 10^8 = \underbrace{5 \cdot 10}_{50} \cdot 10^8 =$$

$$= 5 \cdot 10^9$$

$$\frac{a^m}{a^n} = a^{m-n}$$

LOGARITMES

POTÈNCIA

$$\underset{x}{\overset{\shortparallel}{3}} \to \boxed{2^x} \to 8$$

LOGARITME
en base 2

$$2^{?} = 8 \iff \log_2 8$$

$$\begin{array}{l} 2 \to \\ 4 \to \\ 3 \to \end{array} \boxed{3^x} \begin{array}{l} \to 3^2 = 9 \\ \to 81 \\ \to 27 \end{array}$$

$$3^{?} = 81 \iff \log_3 81 = 4$$

$$3^4 = 81 \checkmark$$

$$\log_3 27 = 3$$

EXEMPLES

(a) $\log_2 4 = 2 \iff 2^{?} = 4 \quad 2^2 = 4$ ✓

(b) $\log_2 16 = 4 \iff 2^{?} = 16$

$2^4 = 16$ ✓

(c) $\log_{10} 1000 = 3 \iff 10^{?} = 1000$

$10^3 = 1000$ ✓

$\log_2 5 =$ no és exacte, queda $2,\ldots$ $\quad 2^{?} = 5 \quad 2^2 = 4$, per tant

$2^3 = 8$, $\log_2 5$ està entre 2 i 3, té xifres decimals

En GENERAL

$$\log_a x = y \iff a^y = x$$

$\log_a X$ $\quad$ a sempre +, $a > 0$

$\log_2 14$, $\log_7 34$, $\log_{\sqrt{2}} 1000$, $\log_{\frac{1}{3}} 4$, $\cancel{\log_{-2} 4}$

$\log_{10} X = \log X$ $\quad$ EX $\log 3 = \log_{10} 3$

10^x $\quad$ e^x

$\boxed{\log}$ $\quad$ $\boxed{\ln}$ $\quad$ $\log 1000 = 3$

$\log_e X$ = logaritmo NEPERIANO = $\ln X$

$e = 2{,}71828\ldots$ $\quad$ Euler

$6^3 = ?$ $\boxed{X^Y}$

6 $\boxed{X^y}$ 3 $\boxed{=}$ 216

10^3 = CALCULADORA

$\boxed{3} \to \boxed{\text{SHIFT}}$ / $\boxed{\text{INV}} \to \overset{10^x}{\boxed{\log}} \to \boxed{=}$ 1000

log 500 = CALCULADORA

$\boxed{500} \to \boxed{\log} \to \boxed{=}$ 2,6989...

$\log 100 = 2$ $\qquad$ $\ln e^3 = 3$ $\qquad$ $\log \sqrt{100} = 1$

$10^? = 100$ $\qquad$ $e^? = e^3$ $\qquad$ $10^? = 10$

$\ln \sqrt[3]{e} = \frac{1}{3}$ $\qquad$ $\log \frac{1}{1000} = -3$

$e^? = e^{\frac{1}{3}}$ $\qquad$ $10^? = \frac{1}{10^3} = 10^{-3}$

$\ln \frac{1}{\sqrt[3]{e}} = -\frac{1}{3}$ $\qquad$ $\boxed{a^{-n} = \frac{1}{a^n}}$

$e^? = \frac{1}{e^{\frac{1}{3}}} = e^{-\frac{1}{3}}$

$$\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = ? \iff 3^{?} = 3^{\frac{1}{2}}$$

$$= \frac{1}{2}$$

$$\log_7 \sqrt[9]{7^5} = \frac{5}{9} \qquad 7^{?} = 7^{\frac{5}{9}}$$

$$\log_8 8^3 = 3 \qquad 8^{?} = 8^{3}$$

$$\log_4 \sqrt[3]{16} = \frac{2}{3} \qquad 4^{?} = 4^{2/3}$$

$$\log_{\frac{1}{2}} 4 = -2 \qquad \left(\frac{1}{2}\right)^{?} = 2^{2}$$

$$? = -2$$

Calcula el valor de x:

a) $\log_x 81 = 2$ $\quad x^{2} = 81$

$x = 9$

b) $\log_5 x = 3$ $\quad 5^{3} = x \Rightarrow x = 125$

c) $\log_3 x = -1$ $\quad 3^{-1} = x$

PROPIETATS dels LOGARITMES

$\log_a a = 1 \Leftrightarrow a^1 = a$

$\log(2+3) \neq \log 2 + \log 3$

$a^m \cdot a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$\left(a^m\right)^n = a^{m \cdot n}$

$$\boxed{\begin{aligned} \log x \cdot y &= \log x + \log y \\ \log \frac{x}{y} &= \log x - \log y \\ \log x^n &= n \cdot \log x \end{aligned}}$$

EXEMPLES

a) Si $\log a = 4$, calcula $\log 10a$

$\log 10 \cdot a = \log 10 + \log a = 1 + 4 = 5$

b) Si $\log b = 5$, calcula

$\log \sqrt[3]{b} = \log b^{1/3} = \frac{1}{3} \cdot \log b = \frac{1}{3} \cdot 5 = \frac{5}{3}$

c) Si $\log a = 4$, $\log b = 5$

CALCULA

$\log 100ab = \log 100 + \log a + \log b$

$= 2 + 4 + 5 = 11$

d) $$\log \frac{a}{1000b} = \log a - \log 1000b =$$

$$= \log a - (\log 1000 + \log b) = 4 - (3+5)$$

$$= 4 - 8 = -4$$

e) $$\log \frac{\sqrt[5]{a}}{\sqrt[7]{b}} = \log \sqrt[5]{a} - \log \sqrt[7]{b} =$$

$$= \log a^{\frac{1}{5}} - \log b^{\frac{1}{7}} = \frac{1}{5}\log a - \frac{1}{7}\log b = \frac{4}{5} - \frac{5}{7} =$$

$$= \frac{28-25}{35} = \frac{3}{35}$$

(f) $$\ln \frac{5\sqrt{e}}{3} = \ln 5\sqrt{e} - \ln 3 =$$

$$= \ln 5 + \ln\sqrt{e} - \ln 3 = \ln 5 + \ln e^{\frac{1}{2}} - \ln 3 =$$

$$= \ln 5 + \frac{1}{2}\underbrace{\ln e} - \ln 3$$

$$\log_a a = 1$$

$$= \boxed{\ln 5 + \frac{1}{2} - \ln 3}$$

(g) Expressa en funció de $\log_a x$, $\log_a y$ i $\log_a z$

en termes de, depenent de

$$\log_a \sqrt[4]{x^2yz} = \log_a (x^2yz)^{\frac{1}{4}} = \frac{1}{4}\log_a (x^2yz) =$$

$$= \frac{1}{4}\cdot\left(\log_a x^2 + \log_a y + \log_a z\right) = \frac{1}{4}\cdot\left(2\log_a x + \log_a y + \log_a z\right)$$

h

Troba el valor de x fent servir la definició de logaritme

$\log_2 32 = x \iff 2^x = 32 \quad \boxed{x=5}$

$\log_4 x = 3 \iff 4^3 = x = 64$

$\log_3 \frac{1}{27} = x = \begin{cases} \log_3 \left(\frac{1}{3}\right)^3 = 3 \cdot \log_3 3^{-1} = 3\cdot(-1) = -3 \\ \log_3 1 - \log_3 27 = 0 - 3 = -3 \end{cases}$

$\log_{\frac{1}{2}} x = 6$

$\iff$

$\frac{1}{64} = \left(\frac{1}{2}\right)^6 = x$

$\log_x 2 = \frac{1}{2}$ $\quad x^{\frac{1}{2}} = 2$

$\sqrt{x} = 2$

$(\sqrt{x})^2 = 2^2 \Rightarrow \boxed{x = 4}$

FULL SOLUCIONAT 32 a), b), c)

a) $\log_3 729 = x \Leftrightarrow 3^x = 729 = 3^6 \Rightarrow \boxed{x=6}$

b) $\log_{\frac{1}{3}} 81 = x \Leftrightarrow \left(\frac{1}{3}\right)^x = 81 \quad (3^{-1})^x = 3^4$

$3^{-x} = 3^4$

$-x = 4 \Rightarrow \boxed{x=-4}$

c) $\log_4 \frac{1}{256} = x \Rightarrow 4^x = \frac{1}{256} = \frac{1}{4^4} = 4^{-4}$

$x = -4$

(34) a), b)

a) $\log 2 = 0{,}301$

$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0{,}301$ $= 0{,}699$

b) $\log \frac{1}{25} = \log 5^{-2} = -2 \cdot \log 5$ $= -2 \cdot 0{,}699 = -1{,}398$

c) $\log \frac{1}{128} = \log 2^{-7} = -7 \cdot \log 2 = -7 \cdot 0{,}301 = -2{,}107$

PROVES d'ACCÉS

Calcula de manera exacta i simplifica si es pot:

$\log 1000 - \ln e^2 = \log 10^3 - \ln e^2$

$3 - 2 = 1$

1. Digueu si és cert o fals i escriviu per què.

a) $\sqrt{3} \in Q$ (nombres racionals) Fals, perquè és irracional

b) $[2,3] = \{x \in R : 2 < x < 3\}$ Fals, perquè hauria de ser $(2,3)$

c) $\sqrt[3]{\sqrt{8}} = \sqrt[5]{8}$ $\sqrt[6]{8} \neq \sqrt[5]{8}$

FALS

d) $\log 1 = \ln 1$ $\log_a 1 = 0$ VERTADER

$0 = 0$