

(b)  $a = 2 \cdot 10^3$        $b = 0,3 \cdot 10^{-2}$       calcula  $a \cdot b$  i escriu-lo en NOTACIÓ CIENTÍFICA .

$$a \cdot b = \underbrace{2 \cdot 10^3}_a \cdot \underbrace{0,3 \cdot 10^{-2}}_b = 0,6 \cdot 10^1 = \underbrace{6 \cdot 10^{-1}}_{0,6} \cdot 10^1$$

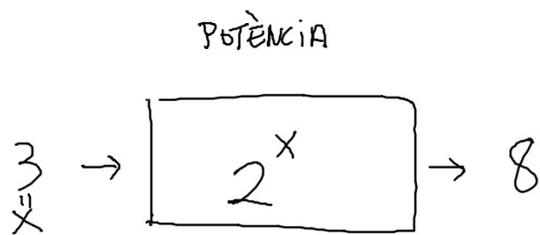
(c)  $a = 200 \cdot 10^3$        $b = 4 \cdot 10^{-5}$        $\frac{a}{b} =$

$$= \frac{200 \cdot 10^3}{4 \cdot 10^{-5}} = 50 \cdot 10^{3-(-5)} = 50 \cdot 10^8 = \underbrace{5 \cdot 10}_{50} \cdot 10^8 =$$

$$= 5 \cdot 10^9$$

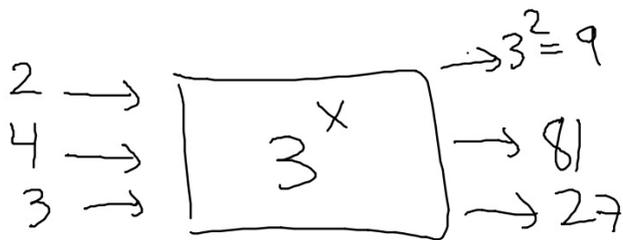
$$\frac{a^m}{a^n} = a^{m-n}$$

## LOGARITMES



$$2^? = 8 \Leftrightarrow \log_2 8$$

LOGARITME  
en base 2



$$3^? = 81 \Leftrightarrow \log_3 81$$
$$3^4 = 81 \checkmark$$

$$\log_3 27 = 3$$

EXEMPLES

(a)  $\log_2 4 = 2 \Leftrightarrow 2^? = 4 \quad 2^2 = 4 \checkmark$

(b)  $\log_2 16 = 4 \Leftrightarrow 2^? = 16$   
 $2^4 = 16 \checkmark$

(c)  $\log_{10} 1000 = 3 \Leftrightarrow 10^? = 1000$   
 $10^3 = 1000 \checkmark$

$\log_2 5 = \text{no és exacte, queda } 2, \dots$       $2^? = 5$       $2^2 = 4$ , per tant  
 $2^3 = 8$       $\log_2 5$  està entre 2 i 3, té xifres decimals

En GENERAL

$\log_a X = Y \Leftrightarrow a^Y = X$

$\log_a X$  a sempre +,  $a > 0$

$\log_2 14$ ,  $\log_7 34$ ,  $\log_{\sqrt{2}} 1000$ ,  $\log_{\frac{1}{3}} 4$ ,  ~~$\log_{-2} 4$~~

$\log_{10} X = \log X$  EX  $\log_3 3 = \log_{10} 3$

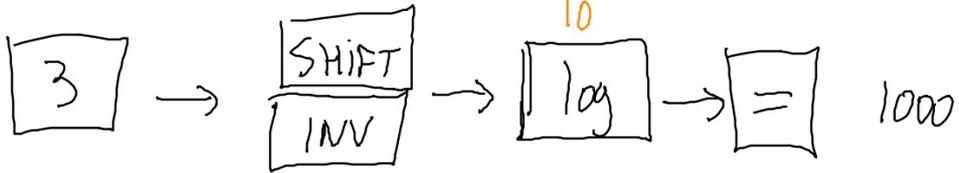
$10^x$   
 $\log$

$e^x$   
 $\ln$

$\log 1000 = 3$

$\log_e X = \text{logaritmo NEPERIANO} = \ln X$   
 $e = 2,71828\dots$  Euler

$10^3 =$  CALCULADORA



$6^3 = ?$

Diagram illustrating the calculator steps for  $6^3$ :

$6 \rightarrow \begin{matrix} X^Y \\ \text{X}^Y \end{matrix} \rightarrow 3 \rightarrow = 216$

$\log 500 =$  CALCULADORA



$$\log 100 = 2$$

$$10^2 = 100$$

$$\ln e^3 = 3$$

$$e^3 = e^3$$

$$\log \sqrt{100} = 1$$

$$10^1 = 10$$

$$\ln \sqrt[3]{e} = \frac{1}{3}$$

$$e^{\frac{1}{3}} = e^{\frac{1}{3}}$$

$$\log \frac{1}{1000} = -3$$

$$10^{-3} = \frac{1}{10^3} = 10^{-3}$$

$$\ln \frac{1}{\sqrt[3]{e}} = -\frac{1}{3}$$

$$e^{-\frac{1}{3}} = \frac{1}{e^{\frac{1}{3}}} = e^{-\frac{1}{3}}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}} = ? \Leftrightarrow 3^? = 3^{\frac{1}{2}}$$

$$= \frac{1}{2}$$

$$\log_7 \sqrt[9]{7^5} = \frac{5}{9} \quad 7^? = 7^{\frac{5}{9}}$$

$$\log_8 8^3 = 3 \quad 8^? = 8^3$$

$$\log_4 \sqrt[3]{16} = \frac{2}{3} \quad 4^? = 4^{\frac{2}{3}}$$

$$\log_{\frac{1}{2}} 4 = -2 \quad \left(\frac{1}{2}\right)^? = 2^2$$

$$? = -2$$

Calcula el valor de x:

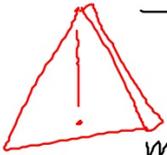
a)  $\log_x 81 = 2$      $x^2 = 81$   
 $x = 9$

b)  $\log_5 x = 3$      $5^3 = x \Rightarrow x = 125$

c)  $\log_3 x = -1$      $3^{-1} = x$

## PROPIETATS dels LOGARITMES

$$\log_a a = 1 \Leftrightarrow a^1 = a$$



$$\log(2+3) \neq \log 2 + \log 3$$

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$\log X \cdot Y = \log X + \log Y$$

$$\log \frac{X}{Y} = \log X - \log Y$$

$$\log X^n = n \cdot \log X$$

### EXEMPLES

(a) Si  $\log a = 4$ , calcula  $\log 10a$

$$\log 10a = \log 10 + \log a = 1 + 4 = 5$$

(b) Si  $\log b = 5$ , calcula  $\log \sqrt[3]{b} = \log b^{1/3} = \frac{1}{3} \cdot \log b = \frac{1}{3} \cdot 5 = \frac{5}{3}$

(c) Si  $\log a = 4$       CALCULA  
 $\log b = 5$        $\log 100ab = \log 100 + \log a + \log b$   
" 2 + 4 + 5 = 11

(d)  $\log \frac{a}{1000b} = \log a - \log 1000b =$   
 $= \log a - (\log 1000 + \log b) = 4 - (3 + 5)$   
 $= 4 - 8 = -4$

(e)  $\log \frac{\sqrt[5]{a}}{\sqrt[7]{b}} = \log \sqrt[5]{a} - \log \sqrt[7]{b} =$  \_\_\_\_\_  
 $= \log a^{\frac{1}{5}} - \log b^{\frac{1}{7}} = \frac{1}{5} \log a - \frac{1}{7} \log b = \frac{4}{5} - \frac{5}{7} =$   
 $= \frac{28 - 25}{35} = \frac{3}{35}$

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$$\begin{aligned}\ln \frac{5\sqrt{e}}{3} &= \ln 5\sqrt{e} - \ln 3 = \\ &= \ln 5 + \ln \sqrt{e} - \ln 3 = \ln 5 + \ln e^{\frac{1}{2}} - \ln 3 = \\ &= \ln 5 + \frac{1}{2} \ln e - \ln 3 \\ \log_a a &= 1 \\ &= \boxed{\ln 5 + \frac{1}{2} - \ln 3}\end{aligned}$$

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Expressa en funció de  $\log_a x$ ,  $\log_a y$  i  $\log_a z$   
en termes de, dependent de

$$\begin{aligned}\log_a \sqrt[4]{x^2 y z} &= \log_a (x^2 y z)^{\frac{1}{4}} = \frac{1}{4} \log_a (x^2 y z) = \\ &= \frac{1}{4} \cdot (\underbrace{\log_a x^2}_{2 \log_a x} + \underbrace{\log_a y}_{\log_a y} + \underbrace{\log_a z}_{\log_a z}) = \frac{1}{4} \cdot (2 \log_a x + \log_a y + \log_a z)\end{aligned}$$

(h)

Troba el valor de  $x$  fent servir la definició de logaritme

$$\log_2 32 = x \Leftrightarrow 2^x = 32 \quad \boxed{x=5}$$

$$\log_4 x = 3 \Leftrightarrow 4^3 = x = 64$$

$$\log_3 \frac{1}{27} = x = \begin{cases} \log_3 \left(\frac{1}{3}\right)^3 = 3 \cdot \log_3 3^{-1} = 3 \cdot (-1) = -3 \\ \log_3 1 - \log_3 27 = 0 - 3 = -3 \end{cases}$$

$$\log_{\frac{1}{2}} x = 6 \quad \left\{ \begin{array}{l} \log_3 1 - \log_3 27 = 0 - 3 = -3 \end{array} \right.$$

$$\frac{1}{64} = \left(\frac{1}{2}\right)^6 = x$$

$$\log_x 2 = \frac{1}{2}$$

$$x^{\frac{1}{2}} = 2$$

$$\sqrt{x} = 2$$

$$(\sqrt{x})^2 = 2^2 \Rightarrow \boxed{x=4}$$

FULL SOLUCIONAT (32) a), b), c)

$$\textcircled{a} \log_3 729 = x \Leftrightarrow 3^x = 729 = 3^6 \Rightarrow \boxed{x=6}$$

$$\textcircled{b} \log_{\frac{1}{3}} 81 = x \Leftrightarrow \left(\frac{1}{3}\right)^x = 81 \quad \left(3^{-1}\right)^x = 3^4$$

$$\textcircled{c} \log_4 \frac{1}{256} = x \Rightarrow 4^x = \frac{1}{256} = \frac{1}{4^4} = 4^{-4} \quad \left. \begin{array}{l} 3^{-x} = 3^4 \\ -x = 4 \end{array} \right\} \Rightarrow \boxed{x=-4}$$

$x=-4$

34 a), b)

a)  $\log 2 = 0,301$

$$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - 0,301$$

b)  $\log \frac{1}{25} = \log 5^{-2} = -2 \cdot \log 5 \quad \underline{= 0,699}$

c)  $\log \frac{1}{128} = \log 2^{-7} \quad \underline{= -2 \cdot 0,699 = -1,398}$   
 $= -7 \cdot \log 2 = -7 \cdot 0,301 = -2,107$

PROVES d'ACCÉS

Calcula de manera exacta i simplifica si es pot:

$$\log 1000 - \ln e^2 = \log 10^3 - \ln e^2$$
$$3 - 2 = 1$$

**1. Digueu si és cert o fals i escriviu per què.**

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a)  $\sqrt{3} \in \mathbb{Q}$  (nombres racionals) Fals, perquè és irracional

b)  $[2,3] = \{x \in \mathbb{R} : 2 < x < 3\}$  Fals, perquè hauria de ser  $(2,3)$

c)  $\sqrt[3]{\sqrt{8}} = \sqrt[5]{8}$   ~~$\sqrt[6]{8} \neq \sqrt[5]{8}$~~   
FALS

d)  $\log 1 = \ln 1$   $\log_a 1 = 0$  VERITADER  
 $0 = 0$